Recent Developments in the Jackiw-Teitelboim Gravity

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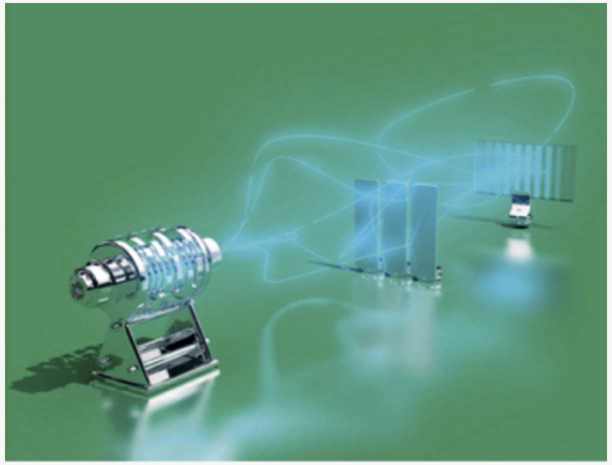
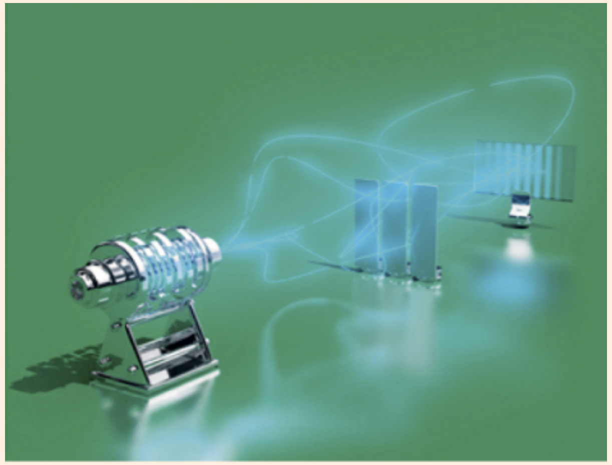
Introduction

-Toward Understanding Quantum Gravity-

Quantum mechanics × General Relativity





Quantum Mechanics

Wave function, partition function....
we must sum over all possible trajectories “path-integral”

General Relativity

Properties of gravity are represented as spacetime geometries

Quantum mechanics × General Relativity

Quantum mechanics

×

General Relativity

Quantum Gravity..?

We must sum over all possible "geometries"..?
Less clear both conceptually and practically....

$$\Psi = \int \mathcal{D}g_{\mu\nu} e^{-I_{\text{grav}}[g_{\mu\nu}]}$$

Current understanding: mostly at semi-classical level

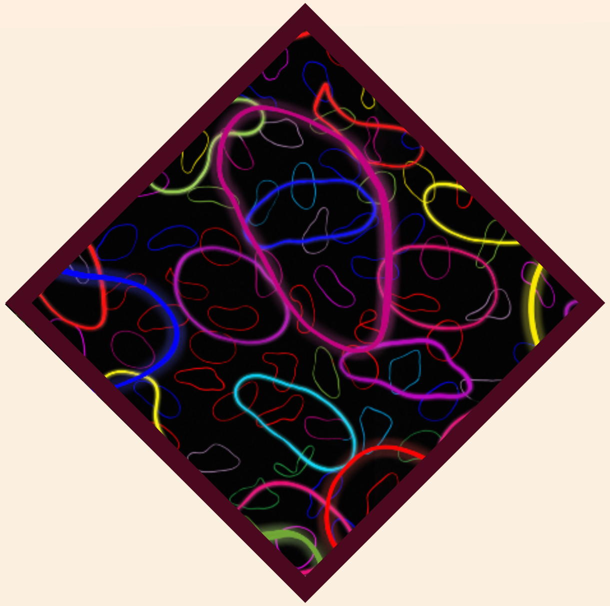
Semi-classical analyses around the black hole background inevitably lead to paradoxes....

Hawking's information loss problem

AMPS's Firewall paradox

Lack of understanding the basic mechanism of quantum gravity

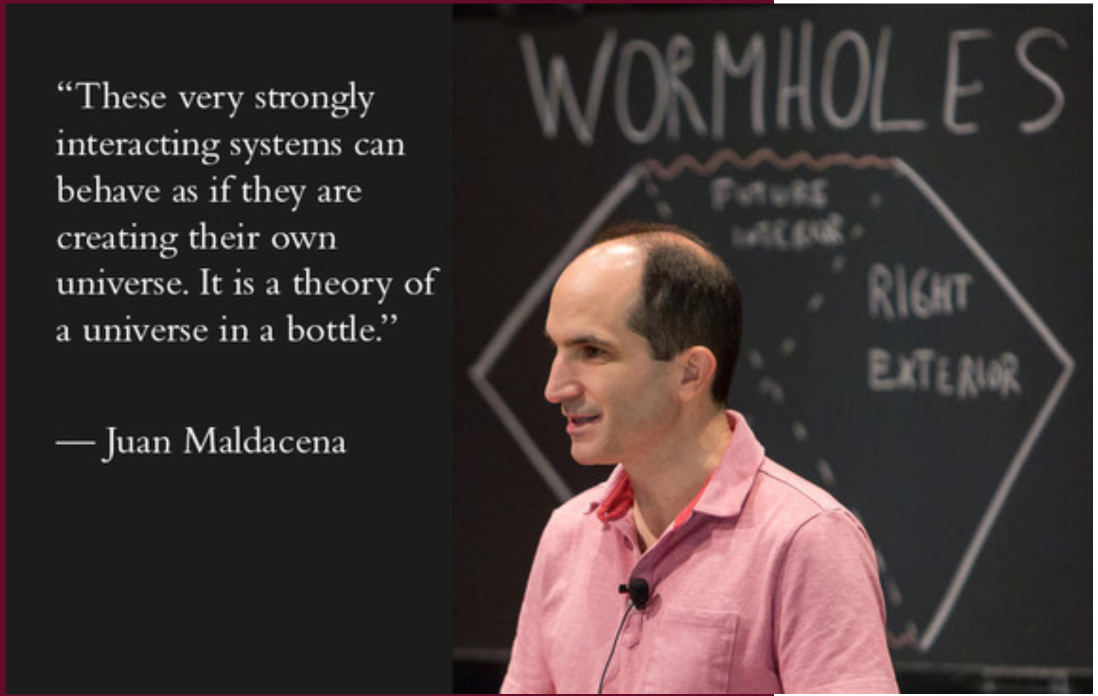
String theory



Miraculously enables us to compute the scattering amplitude with gravitons!

Definition based on the worldsheet picture : perturbation on a fixed background geometry

hard to recapture the concept of spacetime geometry
from the perspective of quantum mechanics



Holographic Principle
-discovery of AdS/CFT

Maldacena '97

Quantum gravity on asymptotically AdS

Conformal field theory on the infinite boundary of AdS

AdS spacetime would be emergent from the quantum degrees of freedom of boundary theory (=CFT)

Can describe (at least outside of) the black holes

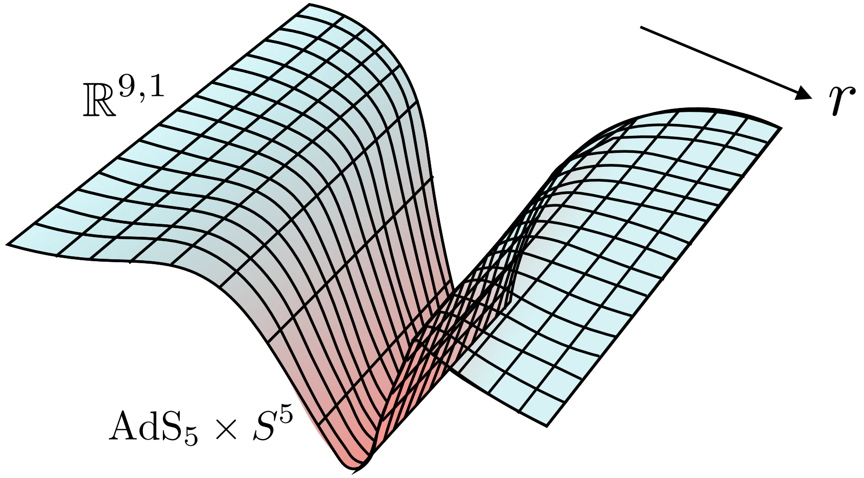
CFT obeys quantum mechanical law and unitary time evolution

Paradox

in AdS_2/CFT_1

AdS_5 from Black 3-Brane

- String theory on a planner black 3-brane geometry



- Decoupling limit: $\ell_p \to 0, \;\; z = \ell_p^2/r$: fixed

  $\to$low-energy effective gravity theory on $AdS_5 \times S^5$

- Energy gap above the vacuum is roughly estimated

$$E \sim 1/V_3^{\frac{1}{3}}$$

V_3 :transverse spatial volume of the brane

$\to$ Energy is finite even after taking the limit $\ell_p \to 0$

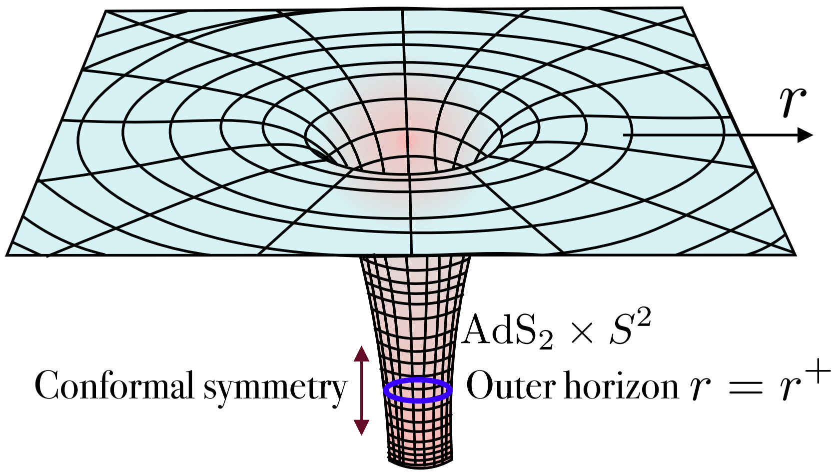
Paradox

in AdS_2/CFT_1

AdS_2 from Black Hole…?

Maldacena-Michelson-Strominger '98

- Gravity theory on the (3+1)-dim near-extremal black hole



$$M \geq Q$$

- Near-horizon limit: $\ell_p \to 0, \ z = Q^2\ell_p^2/(r - r^+)$: fixed

  $\rightarrow$geometry approaches to $AdS_2 \times S^2$

- Energy gap above the vacuum is roughly estimated

$$E \sim 1/(\ell_p Q^3)$$

$\rightarrow$ Energy gap is infinite after taking the limit $\ell_p \to 0$

No dynamics in AdS_2!

Paradox in AdS_2/CFT_1

Exactly at the "AdS_2" (conformal) limit of the extremal black hole, s-wave physics is described by the effective 2d gravity action

$$I_{2d} = \frac{\phi_0}{16\pi G_N}\int \sqrt{g}R + I_{\rm matter}$$

$\phi_0 = 4\pi r_h^2$: horizon area of the extremal black hole

- E-H term is topological→no contribution to E.O.M.

→ Einstein equation sets the stress tensor to zero

$$T_{\mu\nu} = 0$$

No dynamics!

- From CFT_1 point of view, (Virasoro) conformal symmetry enforces the "traceless" condition on the stress tensor.

In 1-dim: $H = T_{00} = 0$

Jackiw-Teitelboim gravity

Move a little away from the strict near-horizon limit
$\rightarrow$conformal symmetry is slightly broken

JT gravity: $I_{JT} = \frac{\phi_0}{16\pi G_N}\int \sqrt{g}R + \frac{1}{16\pi G_N}\int \sqrt{g}\phi(R+2) + I_{\mathrm{matter}}$

$$\phi \ll \phi_0 \Leftrightarrow (r - r_h) \ll r_h$$

Solve the EOMs with asymptotic b.c. : $ds^2|_{\mathrm{bdy}} = \frac{du^2}{\epsilon^2}, \quad \phi|_{\mathrm{bdy}} = \frac{\phi_r}{\epsilon}$

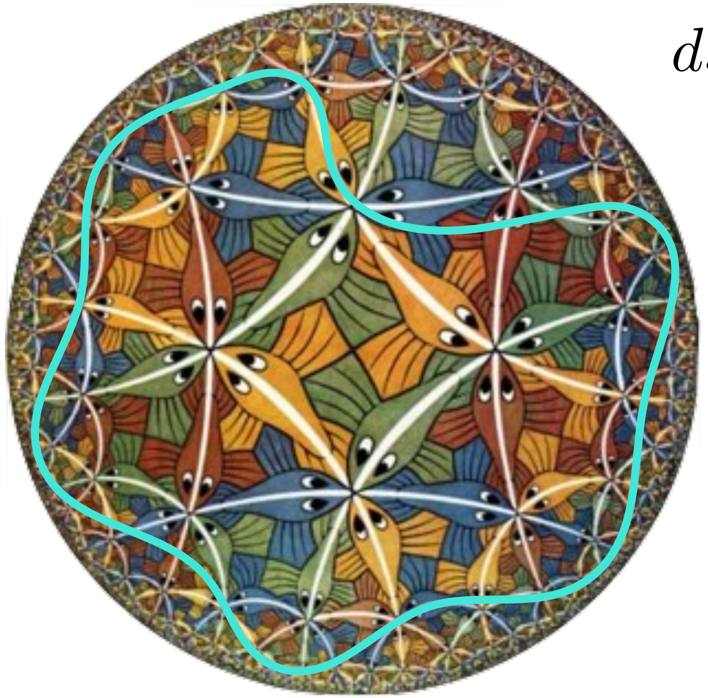
τ: AdS_2 time coordinate (emergent time) u: physical time of the boundary quantum system

EOM for $\phi : R + 2 = 0$ $\rightarrow$ AdS_2 solution

EOM for metric: fixes ϕ

Jackiw-Teitelboim gravity

Solution(s): $ds^2 = d\rho^2 + \sinh^2 \rho d\tau^2\ ,\ \ \phi = \phi_h \cosh \rho$



$$ds^2|_{\mathrm{bdy}} = \frac{du^2}{\epsilon^2}, \qquad \phi|_{\mathrm{bdy}} = \frac{\phi_r}{\epsilon}$$

$$\frac{1}{\epsilon^2} = (\rho'(\tau) + \sinh^2 \rho(\tau)) \left(\frac{d\tau}{du}\right)^2$$

Infinitely many solutions

Conformal symmetry is broken

Jackiw-Teitelboim gravity

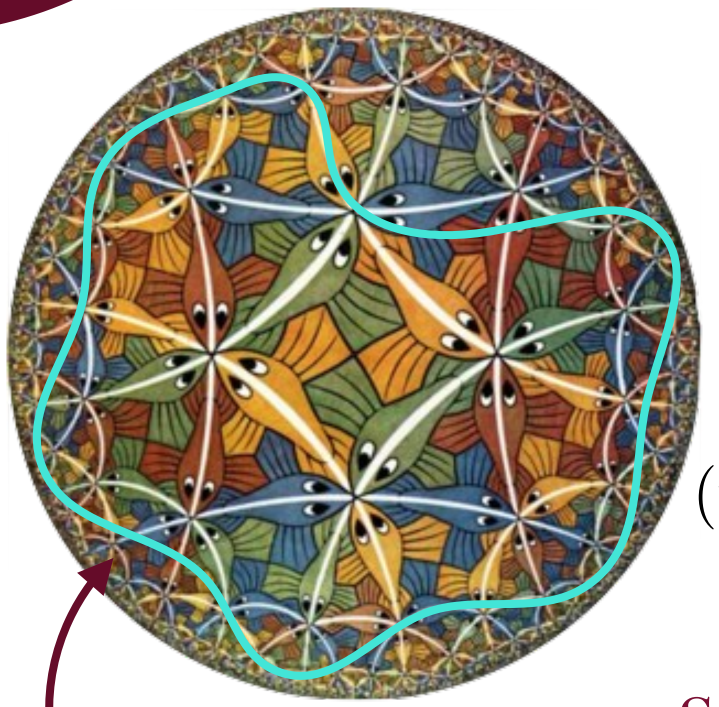
What action are shapes of the boundary governed by?

topological

$$I_{JT} = \frac{\phi_0}{16\pi G_N}\left[\int_{\mathcal{M}} R + 2\int_{\partial\mathcal{M}} K\right] + \frac{1}{16\pi G_N}\left[\int_{\mathcal{M}} \phi(R+2) + 2\int_{\partial\mathcal{M}} \phi(K-1)\right]$$

bulk geometry→zero

depends on the embedding of bdy



$(t(u), z(u))$

$z = \epsilon\sqrt{(t')^2 + (z')^2} = \epsilon t' + O(\epsilon^3)$

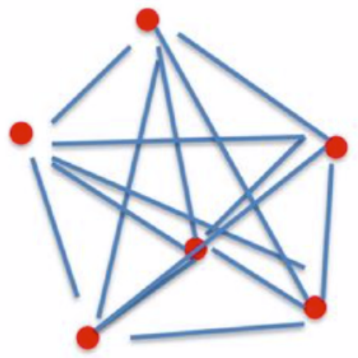
$$ds^2 = \frac{dt^2 + dz^2}{z^2}$$

Schwarzian action $I_{\mathrm{JT}} = \frac{1}{8\pi G_N}\int du\,\phi_r(u)\,\mathrm{Sch}(t,u)$

$$\mathrm{Sch}(t,u) = \left(\frac{t''}{t}\right)' - \frac{1}{2}\left(\frac{t''}{t'}\right)^2$$

Sachdev-Ye-Kitaev model

Quantum mechanical model, only time



$$H = \sum_{ijkl=1}^{N} J_{ijkl}\psi_i\psi_j\psi_k\psi_l \quad \{\psi_i, \psi_j\} = \delta_{ij}$$

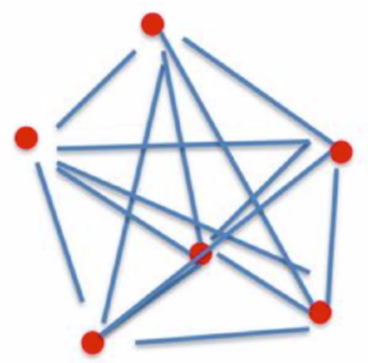
Random coupling: $\langle J_{ijkl}^2 \rangle = J^2/N^3$ $\quad J$: dimension one (relevant) coupling

- Large N limit: the theory solvable
- Maximally chaotic $\rightarrow$ represents black hole physics

In IR: strong coupling limit $\rightarrow$flows (almost) conformal fixed point

Sachdev-Ye-Kitaev model

Quantum mechanical model, only time



$$H = \sum_{ijkl=1}^{N} J_{ijkl}\psi_i\psi_j\psi_k\psi_l \qquad \{\psi_i, \psi_j\} = \delta_{ij}$$

In IR: strong coupling limit →flows (almost) conformal fixed point

- Conformal invariant to leading order in N
- Universal violations of the symmetry to sub-leading orders in $1/N$

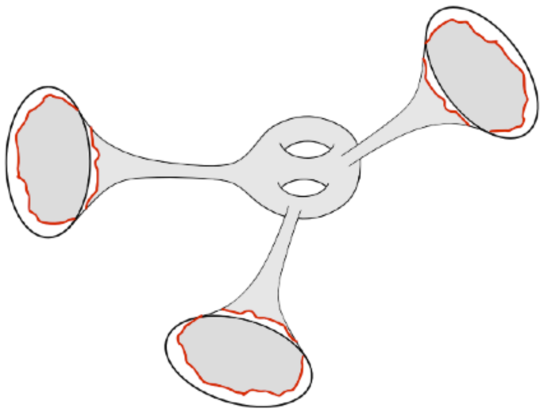
Governed by the Schwarzian action

$$I \propto \frac{N}{J}\int du \mathrm{Sch}(f,u)$$

$f(u)$: reparametrization of time

Partition Functions of the JT model

A remarkable property of the pure JT gravity:
One can calculate the partition function!
[Stanford-Witten, Saad-Shenker-Stanford]



$$I_{JT} = \frac{\phi_0}{16\pi G_N}\left[\int_{\mathcal{M}} R + 2\int_{\partial\mathcal{M}} K\right] + \frac{1}{16\pi G_N}\left[\int_{\mathcal{M}} \phi(R+2) + 2\int_{\partial\mathcal{M}} \phi(K-1)\right]$$

ϕ plays a role of a Lagrange multiplier $\rightarrow$ $R+2=0$

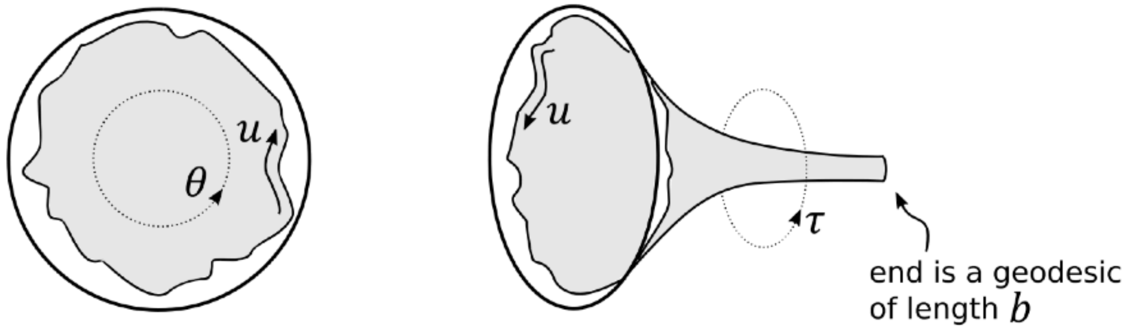
$$Z_{\mathrm{JT}}(\mathcal{M}_{g,n}) = e^{\chi S_0}\int d(\text{ bulk moduli })\int \mathcal{D}(\text{ boundary wiggles })e^{\int_{\partial\mathcal{M}_{g,n}}\phi(K-1)}$$

$\chi = 2-2g-n$

Schwazian action

Partition Functions of the JT model

Simplest examples: $Z_{\rm JT}(\text{disk})$ & $Z_{\rm JT}(\text{trumpet})$



$$ds^2 = d\rho^2 + \sinh^2(\rho)d\theta^2 \quad ds^2 = d\sigma^2 + \cosh^2(\sigma)d\tau^2, \quad \tau \sim \tau + b$$

- No bulk moduli, only integral over a boundary wiggles

$$Z^{\rm disk}_{\rm Sch}(\beta) = \int \frac{d\mu[\theta]}{SL(2,\mathbb{R})} \exp\left[-\frac{\gamma}{2}\int_0^\beta du\left(\frac{\theta''^2}{\theta'^2} - \theta'^2\right)\right]$$

$$Z^{\rm trumpet}_{\rm Sch}(\beta, b) = \int \frac{d\mu[\tau]}{U(1)} \exp\left[-\frac{\gamma}{2}\int_0^\beta du\left(\frac{\tau''^2}{\tau'^2} + \tau'^2\right)\right]$$

Partition Functions of the JT model

From the symplectic measure of the manifold $\mathrm{diff}\left(S^1\right)/SL(2,\mathbb{R})$ (or $\mathrm{diff}\left(S^1\right)/U(1)$), one can pick out the "fermionic" d.o.f.

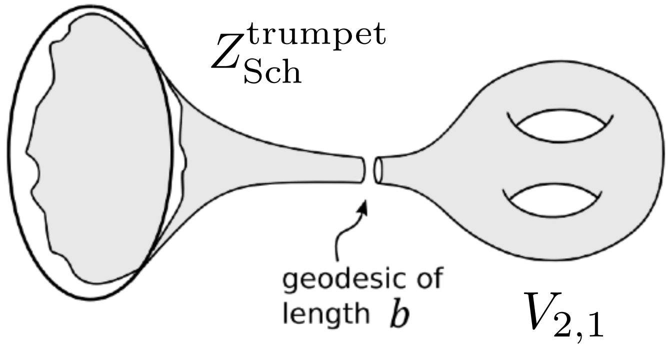
$$\mathcal{Q}(I_{\mathrm{Sch}}(\theta)+I_{\mathrm{measure}}(\theta,\psi))=0$$

$\rightarrow$The integral is one-loop exact! [Stanford-Witten]

$$Z_{\mathrm{Sch}}^{\mathrm{disk}}(\beta)=\frac{\gamma^{3/2}}{(2\pi)^{1/2}\beta^{3/2}}e^{\frac{2\pi^2\gamma}{\beta}}\qquad Z_{\mathrm{Sch}}^{\mathrm{trumpet}}(\beta,b)=\frac{\gamma^{1/2}}{(2\pi)^{1/2}\beta^{1/2}}e^{-\frac{\gamma}{2}\frac{b^2}{\beta}}$$

General geometries?

$$Z_{g,n}\left(\beta_1,\ldots,\beta_n\right)=\int_0^\infty b_1 db\ldots\int_0^\infty b_n db_n V_{g,n}\left(b_1,\ldots,b_n\right)Z_{\mathrm{Sch}}^{\mathrm{trumpet}}\left(\beta_1,b_1\right)\ldots Z_{\mathrm{Sch}}^{\mathrm{trumpet}}\left(\beta_n,b_n\right)$$



$V_{g,n}$: Weil-Petersson volume of the moduli space of hyperbolic Riemann surfaces with genus g and n geodesic boundaries of lengths b_1, ..., b_n.

Partition Functions of the JT model

How can we compute $V_{g,n}$?

→It satisfies "Mirzakhani's recursion relation"

$$W_{g,n}(z_1,\overbrace{z_2,\ldots,z_n}^{J}) = \operatorname*{Res}_{z\to 0}\left\{\frac{1}{(z_1^2-z^2)}\frac{1}{4y(z)}\left[W_{g-1,n+1}(z,-z,J) + \sum_{I\cup I'=J;h+h'=g}^{\prime} W_{h,1+|I|}(z,I)\,W_{h',1+|I'|}(-z,I')\right]\right\}$$

with $W_{g,n}(z_1,\ldots,z_n)=(2\gamma)^{n/2}\int_0^\infty b_1 db_1 e^{-\sqrt{2\gamma}b_1 z_1}\ldots\int_0^\infty b_n db_n e^{-\sqrt{2\gamma}b_n z_n} V_{g,n}(b_1,\ldots,b_n)$ and the "spectral curve" $y=\dfrac{\sin(2\pi z)}{4\pi}$

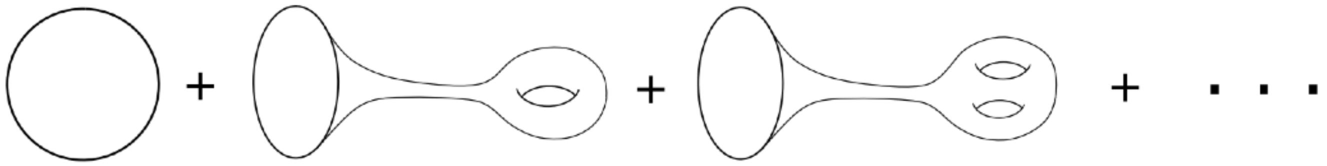
If we take the "density of states" of the Schwarzian theory as an input

$$\rho_0(E)=\frac{\gamma}{2\pi^2}\sinh(2\pi\sqrt{2\gamma E}) \qquad Z_{0,1}(\beta)=\int_0^\infty dE\,\rho_0(E)e^{-\beta E}$$

it agrees with Eynard's "topological recursion" that determines the genus expansion of a matrix integral of the random matrix theory.

Partition Functions of the JT model

From the "topological recursion", one can compute the genus expansion of the JT gravity



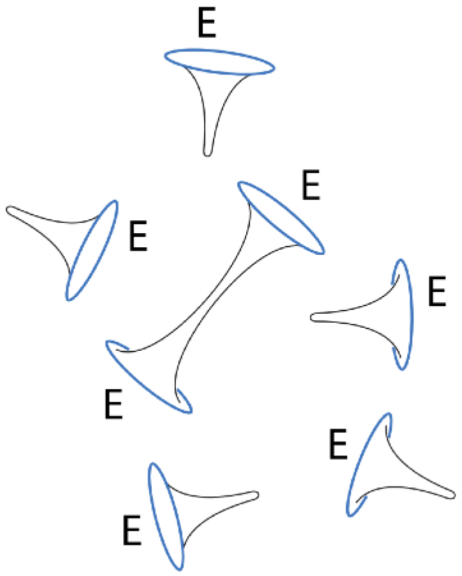
$$\langle Z(\beta)\rangle \simeq e^{S_0} Z^{\text{disk}}_{\text{Sch}}(\beta) + \sum^{\infty} e^{(1-2g)S_0} \int_0^{\infty} bdbV_{g,1}(b) Z^{\text{trumpet}}_{\text{Sch}}(\beta, b)$$

Since $G_N \sim 1/S_0$, expansion parameter $e^{S_0} \sim e^{1/G_N}$

→ non-perturbative splitting and joining of closed JT "baby universes."

The series divergent $Z^{(g)} \sim (2g)!$

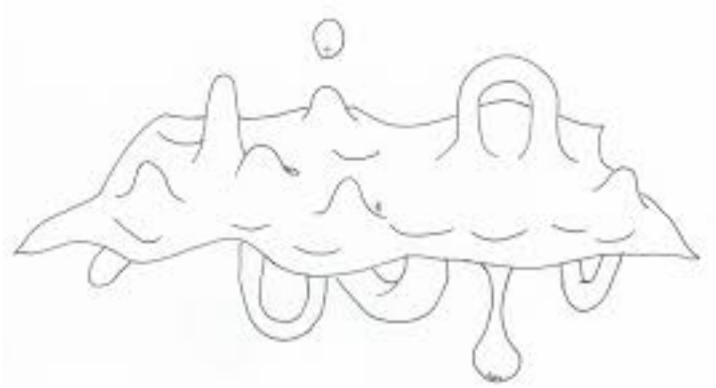
→Non-perturbative completion of this series: "D-branes" described by arbitrary numbers of disconnected universes ending on the brane



What does JT gravity tell us ?

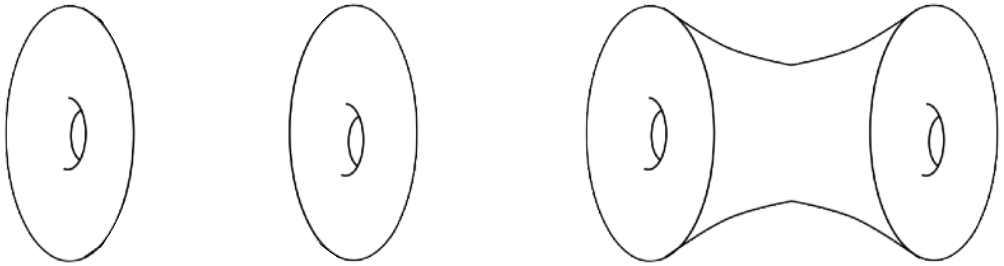
Sum over different topologies in holography?

A problem: quantum gravity seems to have contributions from different topologies



Maldacena& Maoz '04

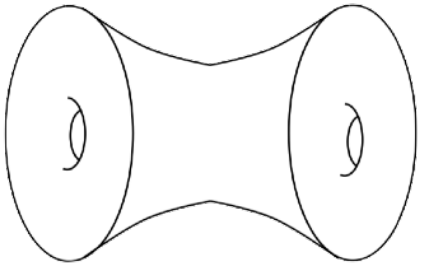
a variety of Euclidean solutions which are asymptotically AdS and which connect *two boundaries*



What does JT gravity tell us ?

Sum over different topology in holography?

However, such two-boundary solutions are problematic in AdS/CFT !

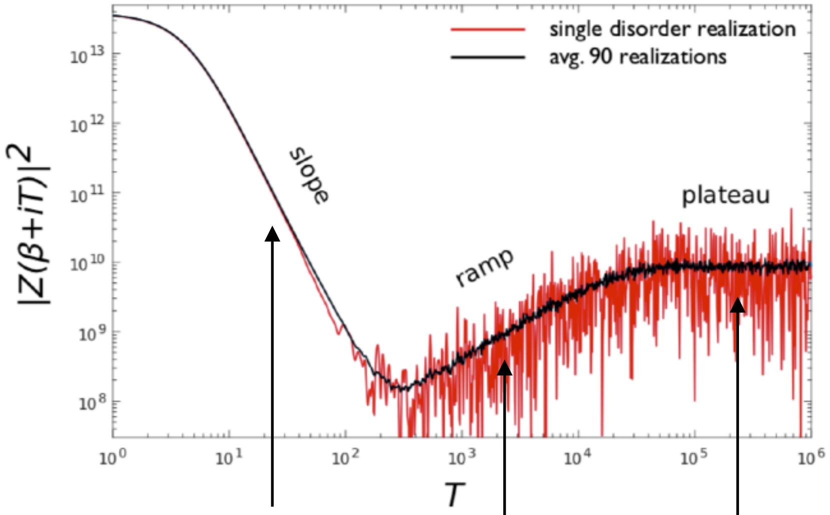


They describe "correlations" between two boundaries

"Correlations" between two CFTs on the different boundaries clearly factorizes!

How about in the JT/ SYK duality?

What does JT gravity tell us ?



slope $\sim \dfrac{e^{2S}}{T^3}$ ramp $\sim T$ plateau $\sim e^S$

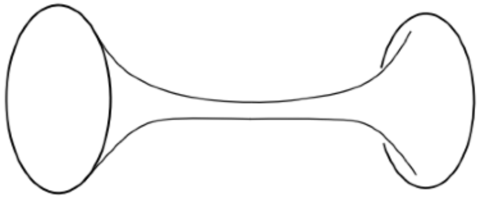
uncorrelated correlated

One can compute "spectral form factor" of SYK model
[Cotlera-Gur-Ari-Hanada-Polchinski-Saad-Shenker-Stanford-Streicher-Tezuka]

$$\langle |Z(iT)|^2 \rangle \equiv \int dJ_{ijkl} e^{-\frac{N^3}{J^2} J_{ijkl}^2} |Z(iT)|^2$$

using two replicas of the system $|Z(iT)|^2 = \underbrace{\mathrm{Tr}\left[e^{-iHT}\right]}_{\text{“}L\text{”}} \cdot \underbrace{\mathrm{Tr}\left[e^{iHT}\right]}_{\text{“}R\text{”}}$

"Ramp" can be reproduced the connected geometry of JT gravity between two boundaries



$$Z_{0,2}(\beta+iT, \beta-iT) \to \frac{1}{2\pi}\frac{T}{\beta_1+\beta_2}$$

What does JT gravity tell us ?

Sum over different topology in holography?

Roughly speaking,

disorder average = emergence of the wormhole geometry,
including non-trivial topologies [cf. Coleman]

Why don't the wormholes contribute for a fixed boundary theory (like d=4 N=4 SYM)?

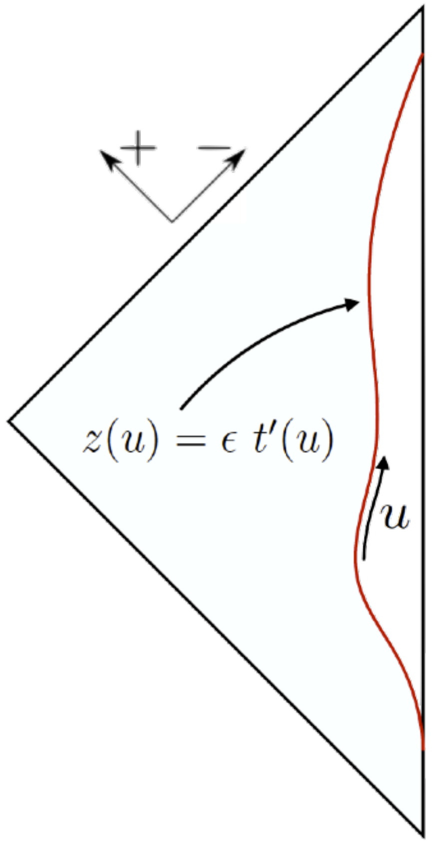
- It is the rule of the game in quantum gravity?
- Miraculously factorizes after summing up all the geometries?

JT gravity + Conformal matter

$$I[g_{\mu\nu},\phi,\chi] = \frac{\phi_0}{16\pi G_N}\int (R+2) + \frac{1}{16\pi G_N}\int \phi(R+2) + \int \phi_b(K-1) + I_{\mathrm{CFT}}[g_{\mu\nu}.\chi]$$

Integrate over ϕ $\rightarrow$ AdS_2 solution $R+2=0$

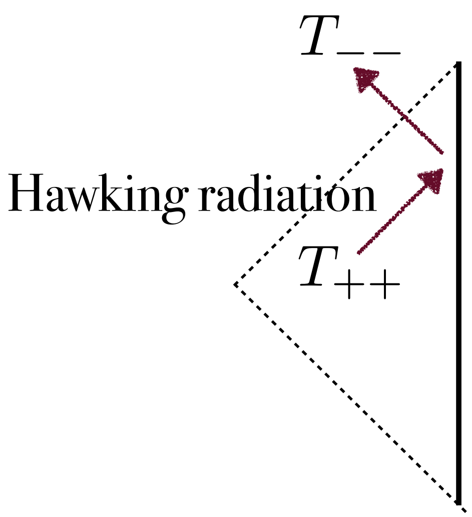
Gravitational dynamics is given by the reparametrization mode $t(u)$



Black hole
information paradox

$I_{\mathrm{CFT}}[g_{\mu\nu},\chi]$ is BCFT on the geometry of AdS_2

JT gravity + Conformal matter



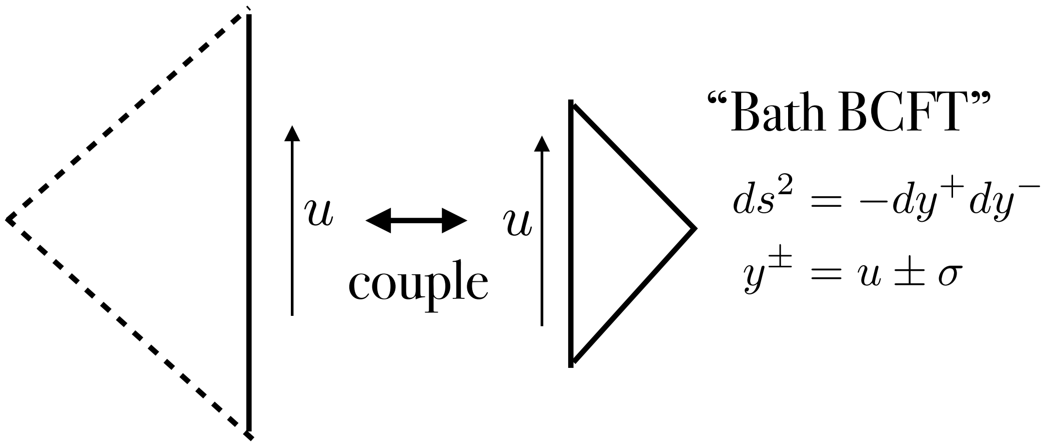
This system leads to perfect reflection at the AdS boundary

$$T_{--} = T_{++}$$

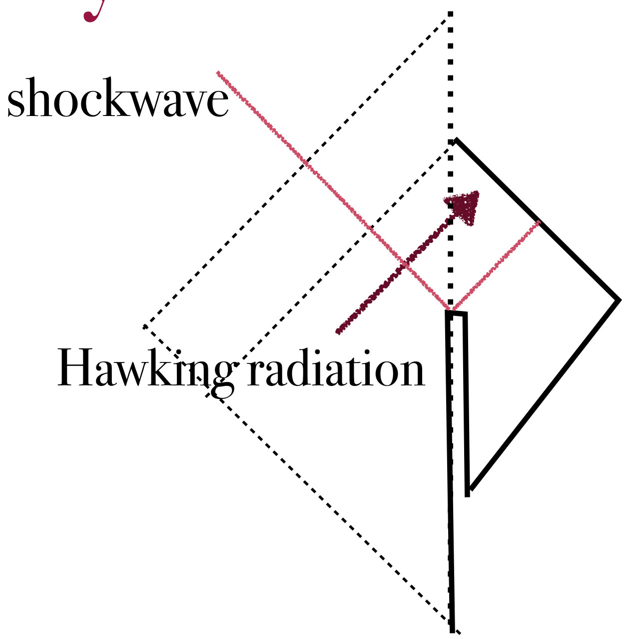
$\rightarrow$ Hawking radiation reflected at the boundary, and the system is in the equilibrium: cannot describe BH evaporation

Black hole information paradox

Simulate an evaporation process by allowing particles to escape from the thermal atmosphere, and evaporation! [Engelsöy-Mertens-Verlinde]



JT gravity + Conformal matter



"Bath BCFT"

$$ds^2 = -dy^+dy^-$$
$$y^{\pm} = u \pm \sigma$$

JT gravity+CFT

$$ds^2 = -\frac{4dx^+dx^-}{(x^+ - x_-)^2}$$
$$x^{\pm} = t \pm z$$

Black hole information paradox

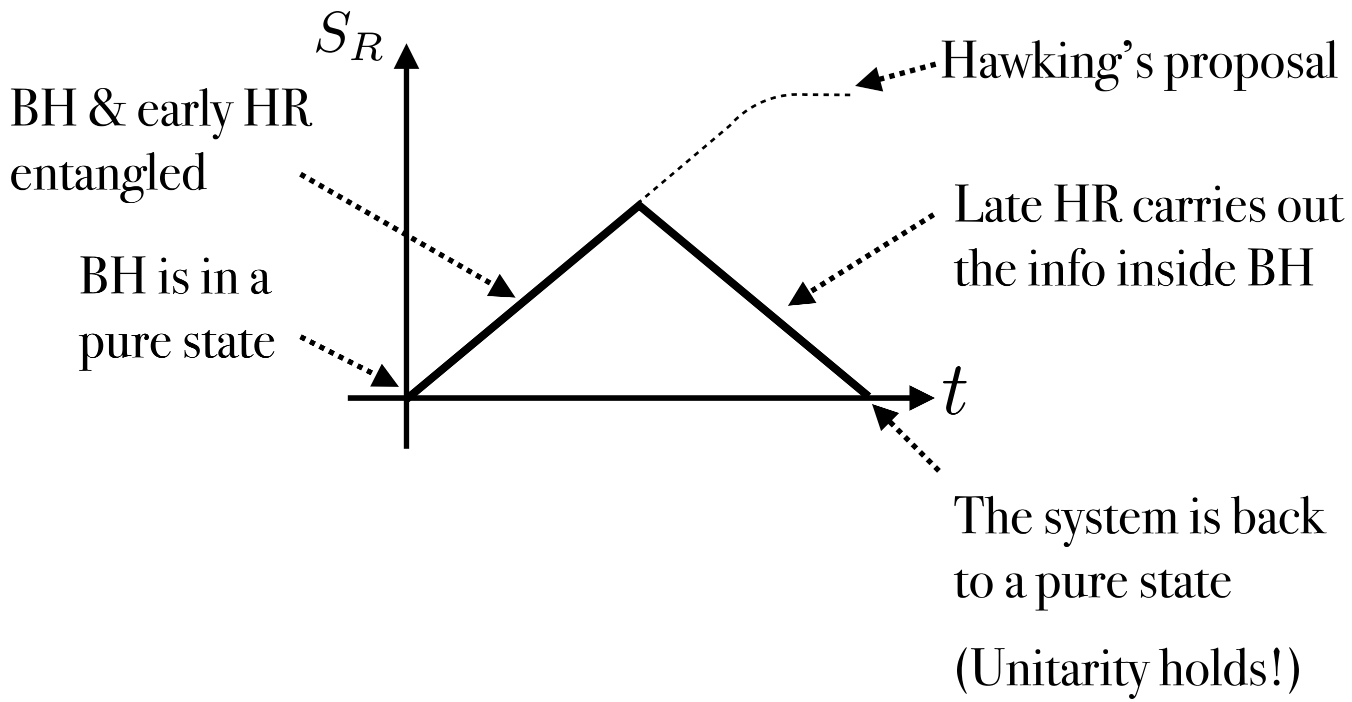
After the initial 'shock' of energy, the energy of the black hole begins to be transferred into the bath via the Hawking radiation

$$\partial_u E(u) = f'(u)^2 (T_{--} - T_{++})$$

Page curve

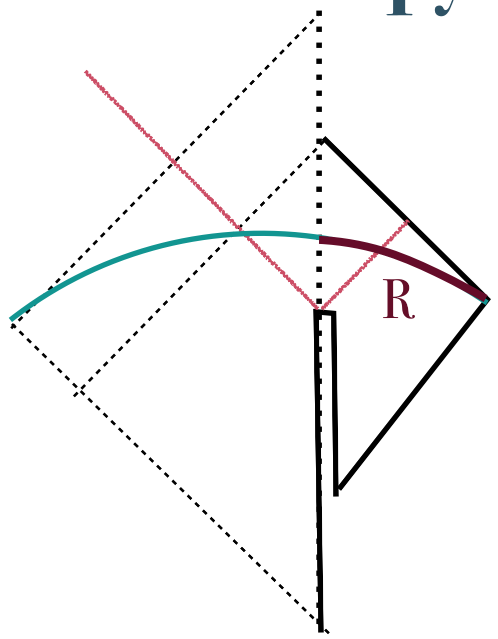
Compute the entropy of the bath (Hawking radiation) S_R as a function of time

What we expect for the unitary evaporation? [Page]



Black hole information paradox

Entanglement entropy of the bath



Making use of the simplicity of the JT model and the 2D CFT, one can compute the entropy of the bath using the replica trick similarly as usual CFT calculation[Almheiri-Engelhardt-Marolf-Maxfield]

$$S_{\mathrm{bath}} \sim \frac{\bar{\phi}}{4G_N} 4\pi T_1 \left(1 - e^{-\frac{k}{2}u}\right)$$

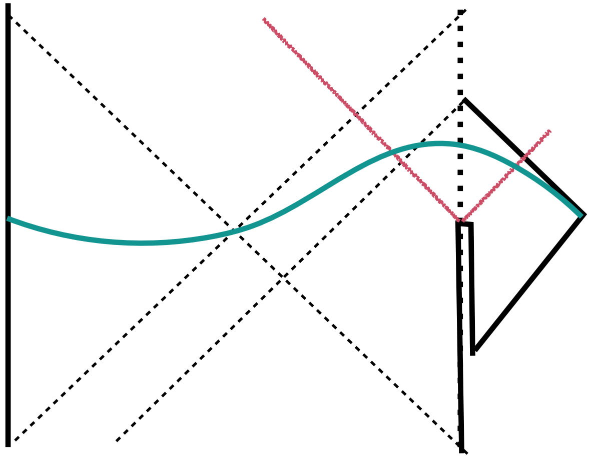
→Agree with the Hawking's argument: entropy thermalizes and contradicts with the unitarity!

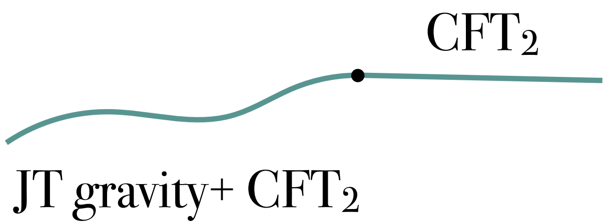
Black hole information paradox

“Holographic description” of JT + CFT

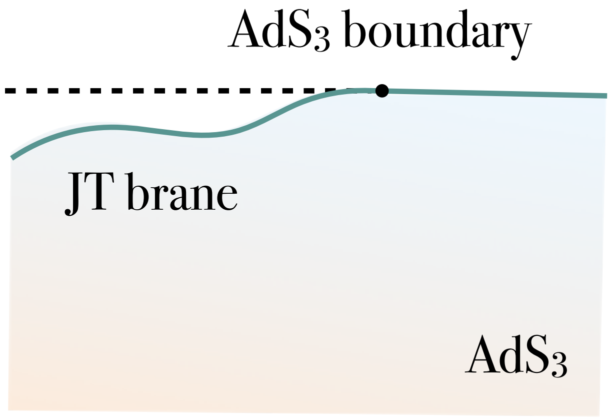
Missing something important?

What if the system has the holographic dual?





2d Gravity



3d Gravity

We replace the CFT$_2$ by its holographic dual [Almheiri-Mahajan-Maldacena-Zhao]

“Holographic description” of JT + CFT

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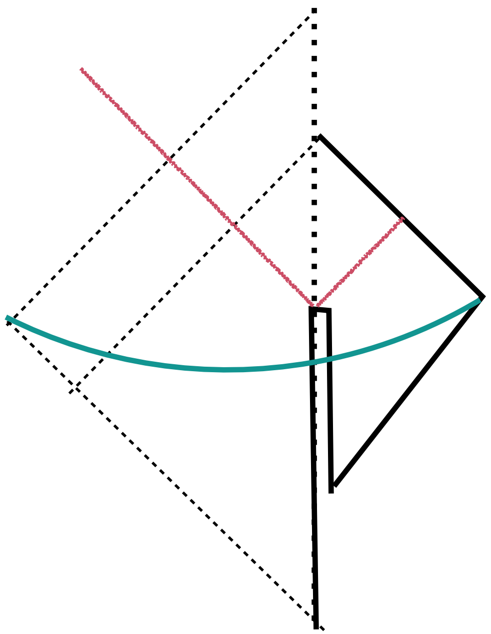

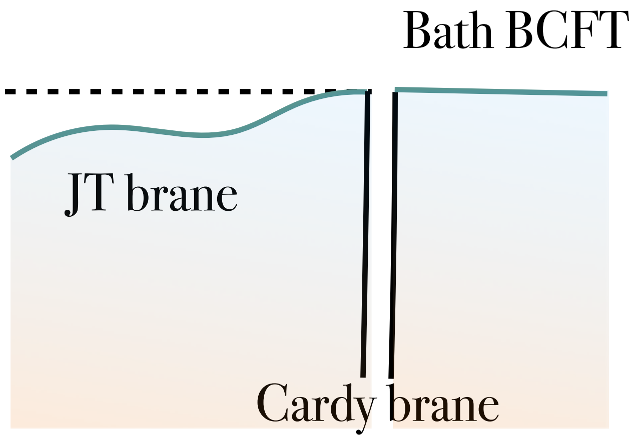

We replace the CFT_2 by its holographic dual [Almheiri-Mahajan-Maldacena-Zhao]

“Holographic description” of JT + CFT

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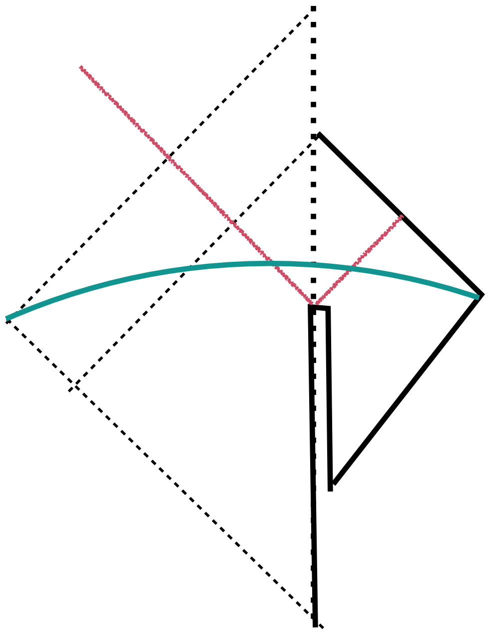
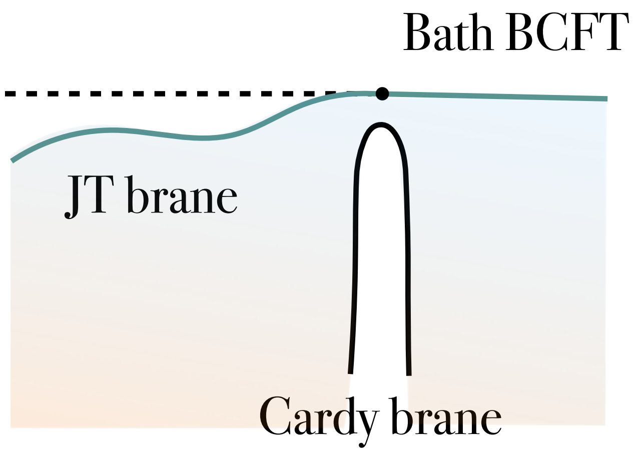


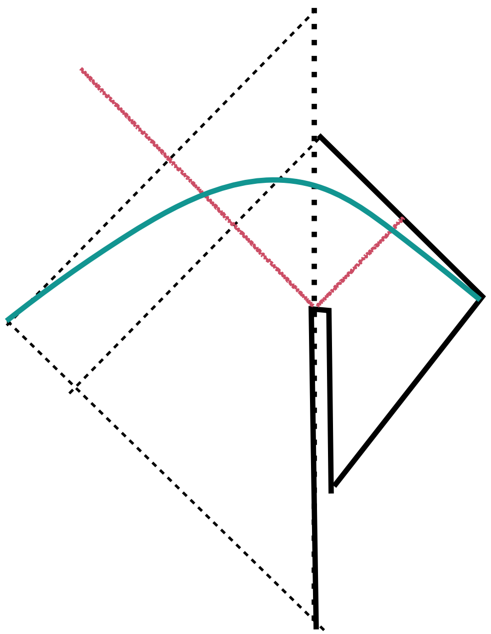


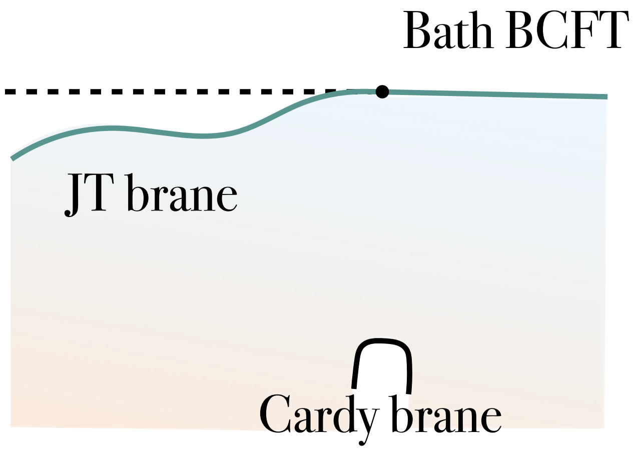
We replace the CFT_2 by its holographic dual [Almheiri-Mahajan-Maldacena-Zhao]

“Holographic description” of JT + CFT

Missing something important?

What if the system has the holographic dual?

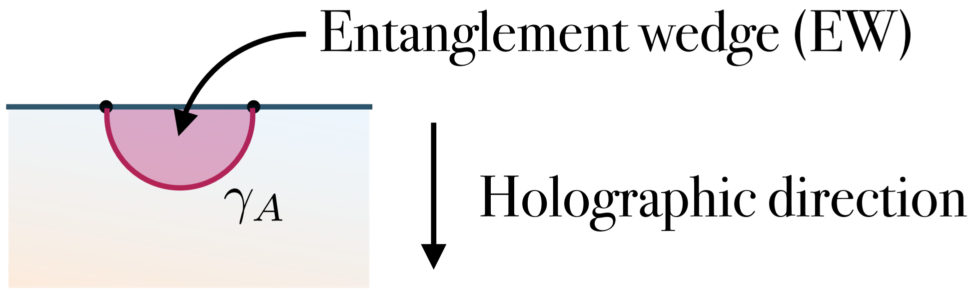




We replace the CFT_2 by its holographic dual [Almheiri-Mahajan-Maldacena-Zhao]

Ryu-Takayanagi formula for the holographic entanglement entropy

How can we holographically compute the entanglement entropy? [Ryu-Takayanagi]



$$S_A = -Tr\rho_A \log \rho_A$$

The entanglement entropy between A and the rest

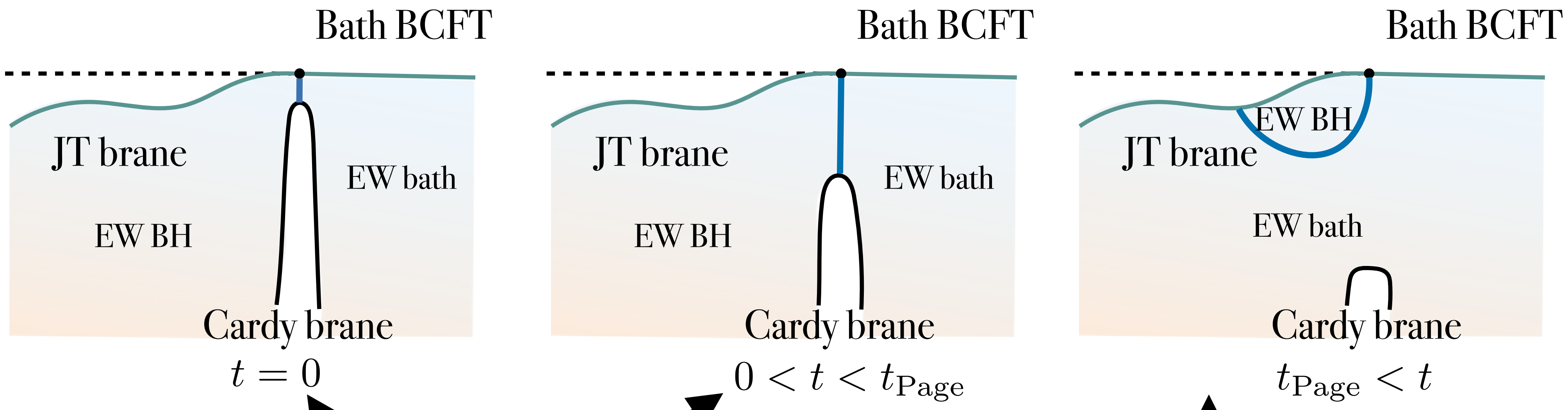
$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N}$$

γ_A is the minimal area surface (codim. =2) such that $\partial A = \partial\gamma_A$ & $A \sim \gamma_A$

- Subregion-subregion duality in AdS/CFT

The information in the "Entanglement wedge" in the bulk is encoded in A of the CFT

“Holographic description” of JT + CFT



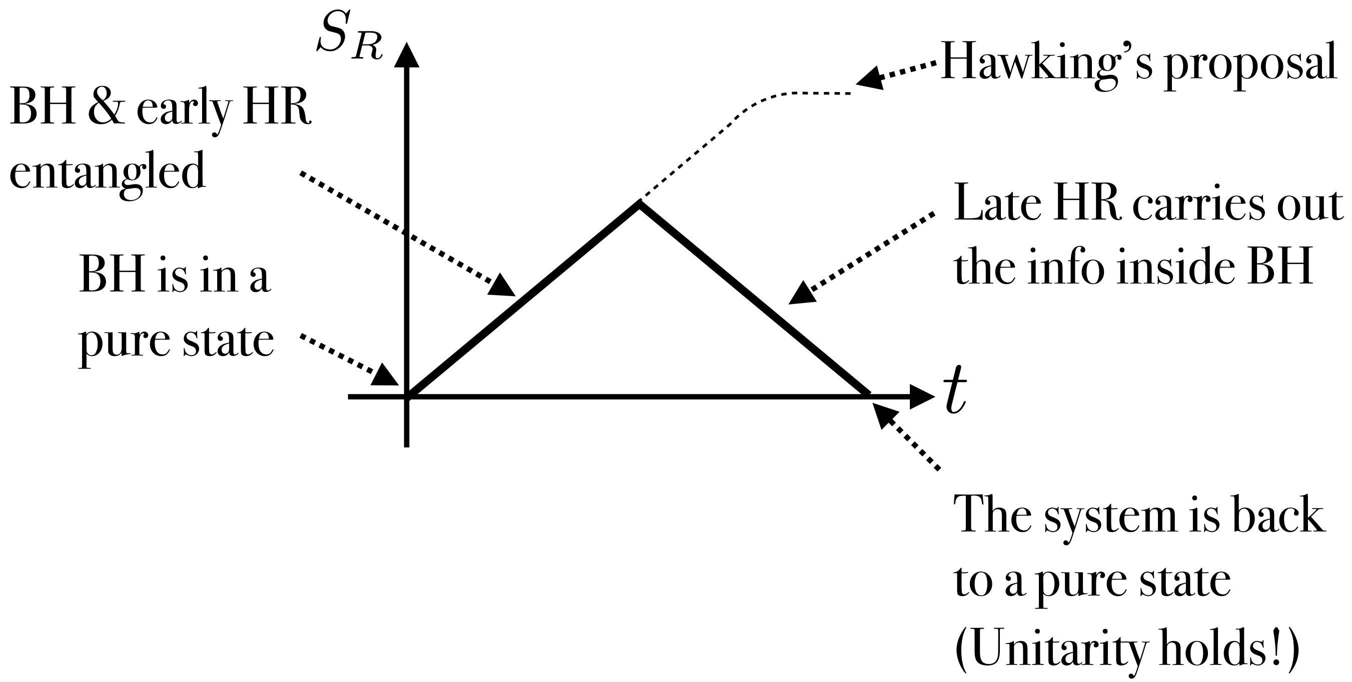
Reproduces $S_{\text{bath}} \sim \dfrac{\bar{\phi}}{4G_N} 4\pi T_1 \left(1 - e^{-\frac{k}{2}u}\right)$

Not included in the usual calculation of the entanglement entropy!

(c): the entanglement wedge connects the black hole interior and the Hawking radiation!
(cf. ER=EPR)

What does it tell us ?

The transition of the Ryu-Takayanagi surface explains the turning point of the Page curve!



What was wrong with the original Hawking's argument..?

Black hole information paradox

Thank you