球面上の超対称ゲージ理論の数値実験
Numerical Experiment of Supersymmetric Gauge Theory on 2-Sphere

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Based on work with K. Ohta, T. Misumi and S. Kamata

Continuum theory

Starting point 2D $N = (2,2)$ SYM theory

$$S_{2d} = \frac{1}{2g^2}\int d^2x \mathrm{Tr}\left\{\frac{1}{2}F_{\mu\nu}^2 + (\mathcal{D}_\mu\phi)(\mathcal{D}^\mu\bar{\phi}) + \frac{1}{4}[\phi,\bar{\phi}]^2 + i\bar{\Psi}\Gamma^\mu\mathcal{D}_\mu\Psi + \frac{i}{2}\bar{\Psi}\Gamma_+[\bar{\phi},\Psi] + \frac{i}{2}\bar{\Psi}\Gamma_-[\phi,\Psi]\right\}.$$

bosons: $A_\mu, \phi, \bar{\phi}$

fermions: $\Psi = \begin{pmatrix}\bar{\psi}\\ \psi\end{pmatrix}$

global symmetries

$U(1)_R$ transformation

$$A_\mu \to A_\mu, \quad \phi \to e^{2i\theta}\phi, \quad \bar{\phi} \to e^{-2i\theta}\bar{\phi}, \quad \psi \to e^{i\theta}\psi, \quad \bar{\psi} \to e^{-i\theta}\bar{\psi}.$$

$U(1)_V$ transformation

$$A_\mu \to A_\mu, \quad \phi \to \phi, \quad \bar{\phi} \to \bar{\phi}, \quad \psi \to e^{i\theta\sigma_2}\psi, \quad \bar{\psi} \to e^{i\theta\sigma_2}\bar{\psi}.$$

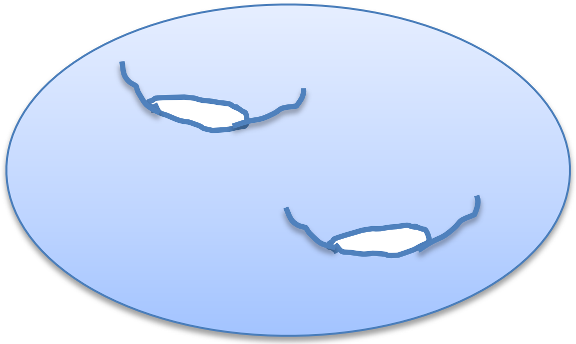
SUSY transformation

$$\delta A_\mu = \bar{\epsilon}^T\gamma_\mu\psi + \epsilon^T\gamma_\mu\bar{\psi}, \qquad \delta\phi = -2i\bar{\epsilon}^T\bar{\psi}, \qquad \delta\bar{\phi} = 2i\epsilon^T\psi,$$

$$\delta\psi = -iF_{12}\sigma_2\epsilon - i(\mathcal{D}_\mu\bar{\phi})\gamma_\mu\bar{\epsilon} + \frac{i}{2}[\phi,\bar{\phi}]\epsilon, \quad \delta\bar{\psi} = -iF_{12}\sigma_2\bar{\epsilon} + i(\mathcal{D}_\mu\phi)\gamma_\mu\epsilon - \frac{i}{2}[\phi,\bar{\phi}]\bar{\epsilon},$$

Derivatives on a curved background

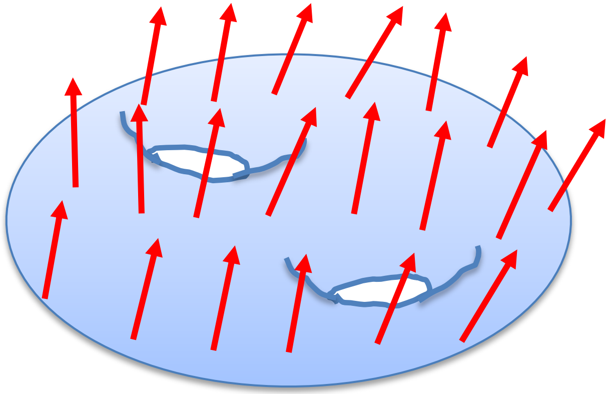
$$\partial_\mu \to \nabla_\mu \qquad \text{spin connection: } \omega_{12} \equiv \omega$$



$$\nabla_\mu \psi = (\partial_\mu + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab})\psi = \partial_\mu\psi - \frac{i}{2}\omega_\mu\sigma_2\psi,$$

$$\nabla_\mu \bar{\psi} = (\partial_\mu - \frac{1}{4}\omega_\mu^{ab}\gamma_{ab})\bar{\psi} = \partial_\mu\bar{\psi} + \frac{i}{2}\omega_\mu\sigma_2\bar{\psi}.$$

Derivatives on a curved background + background $U(1)_V$ field B_μ



$$\partial_\mu\psi \to (\nabla_\mu + \frac{i}{2}B_\mu\sigma_2)\psi = \left(\partial_\mu + \frac{i}{2}(B_\mu - \omega_\mu)\sigma_2\right)\psi,$$

$$\partial_\mu\bar{\psi} \to (\nabla_\mu + \frac{i}{2}B_\mu\sigma_2)\bar{\psi} = \left(\partial_\mu + \frac{i}{2}(B_\mu + \omega_\mu)\sigma_2\right)\bar{\psi},$$

Special background or Topological twisting

Seiberg 2011, 2012

$$B_\mu = \omega_\mu \quad \Rightarrow \quad \begin{aligned} &\left(\partial_\mu + \frac{i}{2}(B_\mu - \omega_\mu)\sigma_2\right)\psi \to \partial_\mu\psi &&: \text{scalar} \\ &\left(\partial_\mu + \frac{i}{2}(B_\mu + \omega_\mu)\sigma_2\right)\bar{\psi} \to (\partial_\mu + i\omega_\mu\sigma_2)\bar{\psi} &&: \text{vector} \end{aligned}$$

Natural renaming in this background

$$\psi \to (\chi,\ \eta/2)^T \text{ : scalar} \qquad \bar{\psi} \to \lambda_\mu \text{ : vector}$$

Continuum action

$$S_{2d} = \frac{1}{2g^2}\int d^2x\sqrt{g}\mathrm{Tr}\left\{(F_{12}/\sqrt{g})^2 + (\mathcal{D}_\mu\phi)\left(\mathcal{D}^\mu\bar{\phi}\right) + \frac{1}{4}[\phi,\bar{\phi}]^2 \right.$$
$$\left. - 2iE^{\mu\nu}\lambda_\mu\mathcal{D}_\nu\chi + i\lambda^\mu\mathcal{D}_\mu\eta + \lambda^\mu[\bar{\phi},\lambda_\mu] - \psi^a[\phi,\psi_a]\right\},$$
$$(E^{\mu\nu} = \varepsilon^{\mu\nu}/\sqrt{g})$$

$U(1)_R$ symmetry

$$\delta A_\mu = 0, \quad \delta\Phi = 2i\theta\Phi, \quad \delta\bar{\Phi} = -2i\theta\bar{\Phi}$$
$$\delta\lambda_\mu = i\theta\lambda_\mu, \quad \delta\eta = -i\theta\eta, \quad \delta\chi = -i\theta\chi.$$

$U(1)_V$ symmetry

$$\delta\psi_a = \theta\varepsilon_{ab}\psi_b,$$
$$\delta\lambda_\mu = -\theta E_{\mu\nu}\lambda^\nu.$$

SUSY transformation of the action

(1) scalar SUSY transformations: $\epsilon_a = \epsilon_a(x) \quad \bar{\epsilon}_\mu = 0$ $\left(C^{1;\mu\nu} = \frac{-1}{\sqrt{g}}\varepsilon^{\mu\nu}, \quad C^{2;\mu\nu} = g^{\mu\nu} \right)$

$$\delta S = \frac{1}{2g^2}\int dx^2 \sqrt{g}(\partial_\mu \epsilon^a) J_a^\mu(x)$$

preserve for $\epsilon_a = \text{const.}$

$$\left(J_a^\mu(x) = 2i\mathrm{Tr}\left\{ E^{\mu\nu}(\lambda C_a)_\nu f + (\mathcal{D}^\mu\phi)\psi_a + \varepsilon_{ab}E^{\rho\nu}(\mathcal{D}_\rho\phi)\psi_b - \frac{i}{2}(\lambda C_a)^\mu[\phi,\bar{\phi}] \right\} \right)$$

(2) vector SUSY transformations: $\epsilon_a = 0 \quad \bar{\epsilon}_\mu = \bar{\epsilon}_\mu(x)$

$$\bar{\delta} S = \frac{1}{2g^2}\int d^2x\sqrt{g}\,(\nabla^\mu\bar{\epsilon}^\nu)\,\bar{J}_{\mu\nu}(x)$$

preserve iff $\bar{\epsilon}_\mu$ is covariantly const.

$$\left(\bar{J}_{\mu\nu}(x) = 2i\mathrm{Tr}\left\{ \frac{i}{2}C_{a;\nu\mu}\psi_a[\phi,\bar{\phi}] - \sigma^1_{ab}C_{b;\mu\nu}\psi_a f - \lambda_\nu\mathcal{D}_\mu\bar{\phi} - \lambda_\mu\mathcal{D}_\nu\bar{\phi} + g_{\mu\nu}\lambda^\rho\mathcal{D}_\rho\bar{\phi} \right\} \right)$$

$$\delta = i\epsilon_1 \tilde{Q} + i\epsilon_2 Q$$

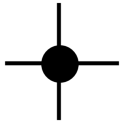
$$\begin{cases} QA_\mu = \lambda_\mu, & Q\lambda_\mu = i\mathcal{D}_\mu\phi, \\ Q\bar{\phi} = \eta, & Q\eta = [\phi, \bar{\phi}], \\ Q\chi = Y, & QY = [\phi, \chi], \\ Q\phi = 0 \end{cases} \qquad \begin{cases} \tilde{Q}A_\mu = \sqrt{g}\varepsilon_{\mu\nu}\lambda^\nu, & \tilde{Q}\lambda_\mu = -i\sqrt{g}\varepsilon_{\mu\nu}\mathcal{D}^\nu\phi, \\ \tilde{Q}\bar{\phi} = 2\chi, & \tilde{Q}\chi = \frac{1}{2}[\phi, \bar{\phi}], \\ \tilde{Q}\eta = -2Y, & \tilde{Q}Y = -\frac{1}{2}[\phi, \eta], \\ \tilde{Q}\phi = 0 \end{cases}$$

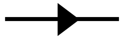
Continuum action in Q-exact form

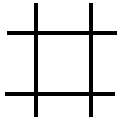
$$S_0 = \frac{1}{2g^2}\int d^2x\sqrt{g}Q\mathrm{Tr}\left\{\frac{1}{4}\eta[\phi, \bar{\phi}] - ig^{\mu\nu}\lambda_\mu\mathcal{D}_\nu\bar{\phi} + \chi\left(Y - \frac{2i}{\sqrt{g}}F_{12}\right)\right\}$$

Observation toward discretized theory

Bosonic fields on lattice

scalar $\phi(x)$ $\Rightarrow$ site variable 

vector $A_\mu(x)$ $\Rightarrow$ link variable 

field tensor $F_{\mu\nu}(x)$ $\Rightarrow$ face variable 

requirement

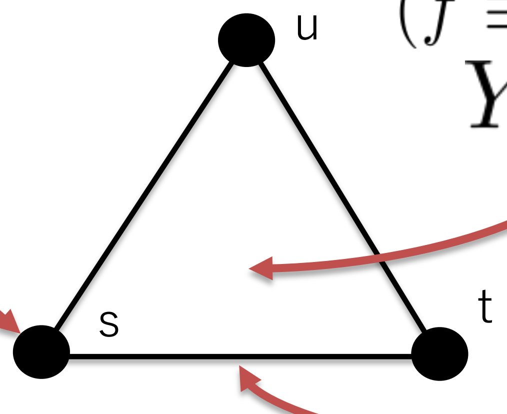
(1) assign bosons on the lattice corresponding to their vector structure

(2) keep Q-symmetry

Fields on lattice

$(f \equiv s\ :\ \text{representative point})$

$\Phi_s, \bar{\Phi}_s, \eta_s$ $\qquad Y_f, \chi_f$



$U_{st}, \Lambda_{st} \ \ (\Lambda_{st} \equiv \lambda_{st} U_{st})$

Discretized action cf) Sugino 2003

$$S = S_S + S_L + S_F \equiv Q \left\{ \sum_{s\in S} \alpha_s \Xi_s + \sum_{\langle st\rangle \in L} \alpha_{\langle st\rangle} \Xi_{\langle st\rangle} + \sum_{f\in F} \alpha_f \Xi_f \right\}$$

$$\begin{cases} \Xi_s \equiv \dfrac{1}{2g_0^2} \mathrm{Tr}\left\{ \dfrac{1}{4} \eta_s [\Phi_s, \bar{\Phi}_s] \right\}, \\ \Xi_{\langle st\rangle} \equiv \dfrac{1}{2g_0^2} \mathrm{Tr}\left\{ -i\lambda_{st} \left(U_{st} \bar{\Phi}_t U_{st}^{-1} - \bar{\Phi}_s \right) \right\}, \\ \Xi_f \equiv \dfrac{1}{2g_0^2} \mathrm{Tr}\left\{ \chi_f \left(Y_f - i\beta_f \mu(U_f) \right) \right\}, \qquad \left(U_f \equiv \prod_{i=1}^{n} U_{s_i s_{i+1}} \right) \end{cases}$$

$$\begin{aligned} & Q\Phi_s = 0, && \\ & Q\bar{\Phi}_s = \eta_s, && Q\eta_s = [\Phi_s, \bar{\Phi}_s], \\ & QU_{st} = i\lambda_{st} U_{st}, && Q\lambda_{st} = i \left(U_{st} \Phi_t U_{st}^{-1} - \Phi_s + \lambda_{st} \lambda_{st} \right), \\ & QY_f = [\Phi_f, \chi_f], && Q\chi_f = Y_f. \end{aligned}$$

Topological information is preserved on lattice

(1) The same localization technique with the continuum theory works also on lattice

<u>non-trivial fixed point equations</u>

$$\Phi_s U_{st} - U_{st}\Phi_t = 0$$
$$\mu(U_f) = 0$$

$\Rightarrow$

<u>1-loop contribution is exact</u>

$$Z \sim \frac{(\Delta(\phi)^2)^{\#(\mathrm{sites})} \times (\Delta(\phi))^{\#(\mathrm{faces})}}{(\Delta(\phi))^{\#(\mathrm{links})} \times (\Delta(\phi))^{\#(\mathrm{sites})}}$$
$$= \prod_{i<j}(\phi_i - \phi_j)^{\chi}$$

Euler characteristic !

(2) The $U(1)_R$ anomaly appears in the measure

$$\mathcal{D}\vec{B} = (\prod_{s=1}^{N_S} \mathcal{D}\Phi_s \mathcal{D}\bar{\Phi}_s)(\prod_{l=1}^{N_L}\mathcal{D}U_l)(\prod_{f=1}^{N_F}\mathcal{D}Y_f)$$: $U(1)_R$ neutral

$$\mathcal{D}\vec{F} = (\prod_{s=1}^{N_S}\mathcal{D}\eta_s)(\prod_{l=1}^{N_L}\mathcal{D}\lambda_l)(\prod_{f=1}^{N_F}\mathcal{D}\chi_f)$$
$$\to \left(\mathcal{D}\vec{\mathcal{F}}\right) e^{i(N_S - N_L + N_F)(N^2-1)\alpha}$$

<u>$U(1)_R$ symmetry</u>

$$\Phi \to e^{2i\alpha}\Phi, \quad \bar{\Phi} \to e^{-2i\alpha}\bar{\Phi}, \quad A_\mu \to A_\mu,$$
$$\eta \to e^{-i\alpha}\eta, \quad \lambda_\mu \to e^{i\alpha}\lambda_\mu, \quad \chi \to e^{-i\alpha}\chi.$$

Natural question : Can we take the continuum limit ?

YES! from power counting point of view

(1) Tree level continuum limit reproduces the continuum action:

$$S = S_S + S_L + S_F \equiv Q \left\{ \sum_{s\in S} \alpha_s \Xi_s + \sum_{\langle st\rangle \in L} \alpha_{\langle st\rangle} \Xi_{\langle st\rangle} + \sum_{f\in F} \alpha_f \Xi_f \right\}$$

$$\boxed{\begin{array}{l} a^2 \equiv vol(\Sigma_g)/N_F \qquad a^2 A_f = \int_{\sigma_f} d^2x \sqrt{g(x)}\,, \qquad e^{\mu}_{st} \equiv \frac{1}{a}(x^{\mu}_t - x^{\mu}_s) \\ \alpha_s = \sum_{f\in F_s} \frac{A_f}{|S_f|}\,, \quad \alpha_f = A_f\,, \quad \beta_f = \frac{1}{A_f}\,, \quad \sum_{\langle st\rangle \in L_f} \alpha^f_{\langle st\rangle} e^{\mu}_{st} e^{\nu}_{st} = A_f g^{\mu\nu}(x_f)\,. \end{array}}$$

(2) There is no Q, $U(1)_R$ and gauge-invariant radiative correction which spoils the geometry.

$$\left(\underbrace{\frac{a^{p-4}}{g^2}}_{\text{tree}} + \underbrace{c_1 p^{p-2}}_{\text{1-loop}} + \underbrace{c_2 a^p g^2}_{\text{2-loop}} + \cdots \right) \int d^2x \sqrt{g}\, \mathcal{O}_p(x) \quad \Longrightarrow \quad \mathcal{O} \sim \mathcal{B} \text{ or } \mathcal{B}^2$$

It is expected that the continuum theory will be obtained by simply taking $a \to 0$.

We have to check it non-perturbatively.

(It gives a “definition” of the SUSY theory on curved background!)

What should we check?

The $U(1)_V$ symmetry and $\tilde{Q}$-symmetry are broken by discretization.

➡ They must be restored in the continuum limit.

fact These symmetries are related with each other in the continuum limit.

$$J_V^\mu = 2i\mathrm{Tr}\left\{\varepsilon_{ab}(\lambda C_a)_\mu \psi_b\right\}.$$

$$J_a^\mu(x) = 2i\mathrm{Tr}\left\{E^{\mu\nu}(\lambda C_a)_\nu f + (\mathcal{D}^\mu\phi)\psi_a + \varepsilon_{ab}E^{\rho\nu}(\mathcal{D}_\rho\phi)\psi_b - \frac{i}{2}(\lambda C_a)^\mu[\phi,\bar{\phi}]\right\}$$

$$Q_a J_V^\mu(x) = \varepsilon_{ab} J_b^\mu(x)$$

$$dJ_Q = 0$$

$$dJ_V = 0 \quad \Rightarrow \quad dJ_{\tilde{Q}} = 0$$

It is sufficient to check the $U(1)_V$ symmetry

Model

(M,N)-polygon decomposition of S^2

parameter tuning

$$ds^2 = R^2(d\theta^2 + \cos^2\theta\, d\varphi^2) \qquad (-\pi/2 < \theta < \pi/2)$$

$$a^2 = \frac{8\pi R^2}{M(N-2)+4}$$

$$\alpha_{s=1\cdots M} = \alpha_{s=\frac{MN}{2}-M+1\cdots\frac{MN}{2}} = \frac{\pi R^2}{Ma^2}\left(2 - \cos\frac{\pi}{N} - -\cos\frac{3\pi}{N}\right)$$

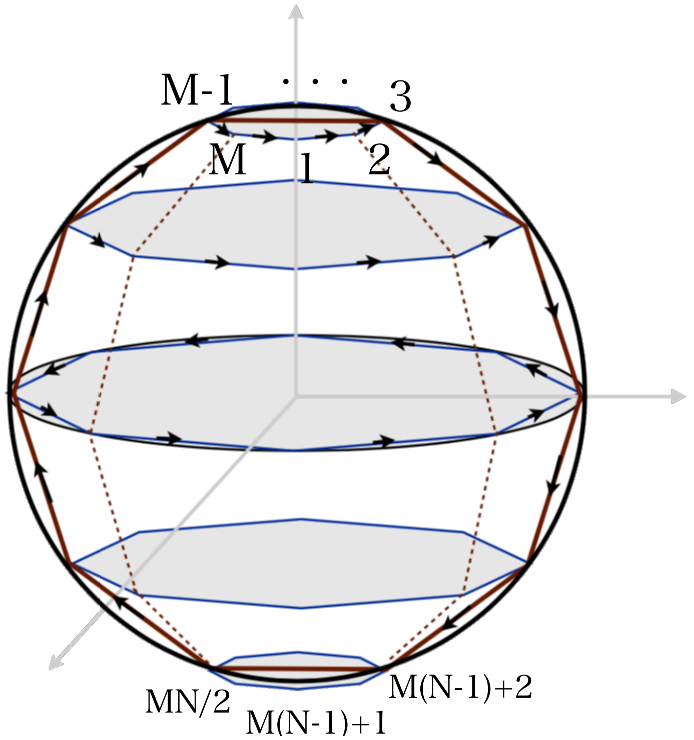
$$\alpha_{s=nM+i} = \frac{\pi R^2}{Ma^2}\left(\cos\frac{2n-1}{N}\pi - \cos\frac{2n+3}{N}\pi\right)$$

$$\alpha_{l=(n-1)M+i} = \frac{M}{\pi}\sin\frac{\pi}{N}\left(\sum_{k=1}^{n-1} - \sum_{k=n}^{N/2-1}\right)\frac{(-1)^{n+k+1}}{\sin\frac{2k\pi}{N}}$$

$$\alpha_{l=MN/2+(n-1)M+i} = \frac{N^2}{2\pi M}\left(\cos\frac{2n-1}{N}\pi - \cos\frac{2n+1}{N}\pi\right)$$

$$\alpha_{f=(n-1)M+i} = \beta^{-1}_{f=(n-1)M+i} = \frac{2\pi R^2}{a^2}\left(\cos\frac{2n-1}{N}\pi - \cos\frac{2n+1}{N}\pi\right)$$

$$\alpha_{f=1,\,M(N-2)/2+2} = \beta^{-1}_{f=1,\,M(N-2)/2+2} = \frac{2\pi R^2}{a^2}\left(1 - \cos\frac{\pi}{N}\right)$$



site	$N \times M/2$
link	$N \times M/2 + M \times (N-2)/2$
face	$2 + M \times (N-2)/2$

Anomaly-phase-quench method

vev in the continuum theory

U(1) charge: $(N^2-1)\chi_h$

$$\langle\mathcal{O}\rangle \equiv \frac{1}{Z_q}\int \mathcal{D}\vec{\mathcal{B}}\mathcal{D}\vec{\mathcal{F}}\;\mathcal{O}\,e^{-S_b-S_f} = \frac{1}{Z_q}\int \mathcal{D}\vec{\mathcal{B}}\;\mathcal{O}\,\mathrm{Pf}(D)\,e^{-S_b}$$

phase quench method in usually used in Monte Carlo method

U(1) charge: ZERO

$$\langle\mathcal{O}\rangle^q \equiv \frac{1}{Z_q}\int \mathcal{D}\vec{\mathcal{B}}'\;\mathcal{O}\,|\mathrm{Pf}(D)|\,e^{-S_b}$$

NOT A GOOD APPROXIMATION

philosophy of the phase quench

Ignore only the artificial phase coming from the discretization

Observation

$$\mathrm{Pf}(D) = |\mathrm{Pf}(D)|e^{i\theta_A + i\theta}$$

1. $\mathrm{U}(1)_{\mathrm{R}}$ phase θ_A
2. lattice artifact θ

We should ignore only θ

Compensator

Kamata-Misumi-Ohta-S.M. 2016

$\mathcal{A}$: an operator with

- $Q\mathcal{A} = 0$
- $[\mathcal{A}] = -(N^2 - 1)\chi_h$
- $\mathcal{A} \equiv |\mathcal{A}| e^{-i\theta_A}$

anomaly-phase-quench method

$$\langle \mathcal{O} \rangle^{\hat{q}} \equiv \langle \mathcal{O}\, e^{i\theta_A} \rangle^q = \frac{1}{Z_q} \int \mathcal{D}\vec{\mathcal{B}} \mathcal{O} |Pf(\mathcal{D})| e^{i\theta_A}$$

trace type

$$\mathcal{A}_{\mathrm{tr}} = \frac{1}{N_S} \sum_{s=1}^{N_S} \left(\frac{1}{N_c} \mathrm{Tr}\,(\Phi_s)^2 \right)^{-\frac{N_c^2-1}{4}\chi_h}$$

determinant type

$$\mathcal{A}_{\mathrm{det}} = \frac{1}{N_S} \sum_{s=1}^{N_S} (\mathrm{Det}\Phi_s)^{-\frac{N_c^2-1}{2N_c}\chi_h}$$

Izykson-Zuber type

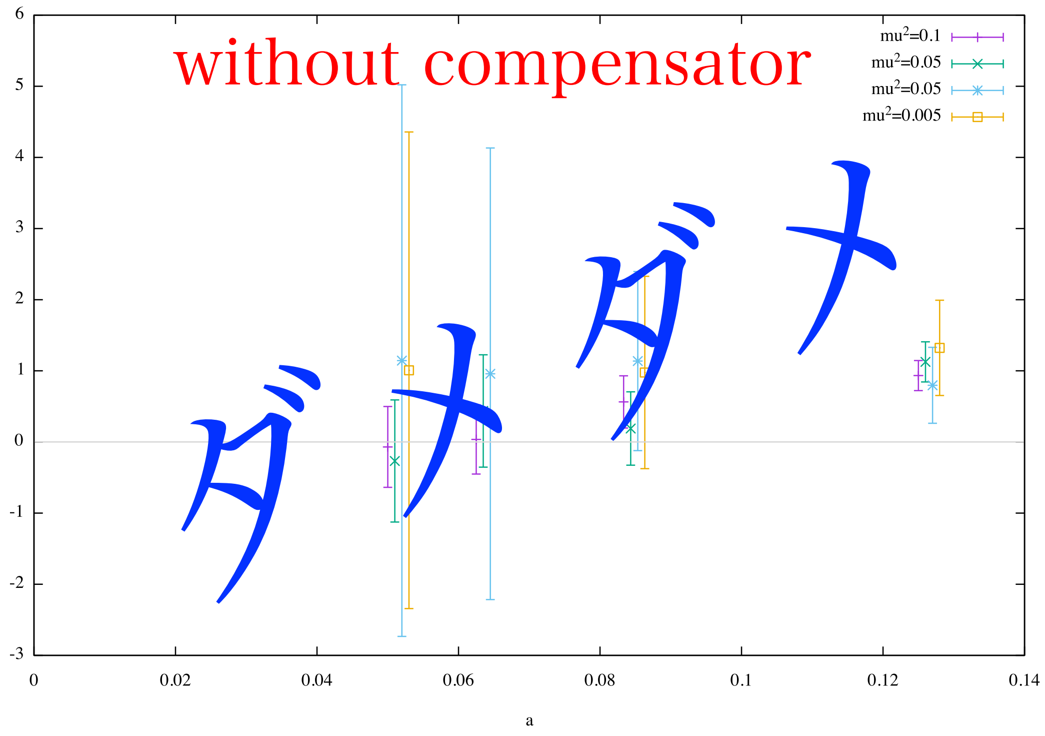
$$\mathcal{A}_{\mathrm{IZ}} = \frac{1}{N_l} \sum_{l=1}^{N_l} \left(\frac{1}{N_c} \mathrm{Tr} \left(2\Phi_{\mathrm{org}(l)} U_l \Phi_{\mathrm{tip}(l)} U_l^\dagger + \lambda_l \lambda_l (U_l \Phi_{\mathrm{tip}(l)} U_l^\dagger + \Phi_{\mathrm{org}(l)}) \right) \right)^{-\frac{N_c^2-1}{4}\chi_h}$$

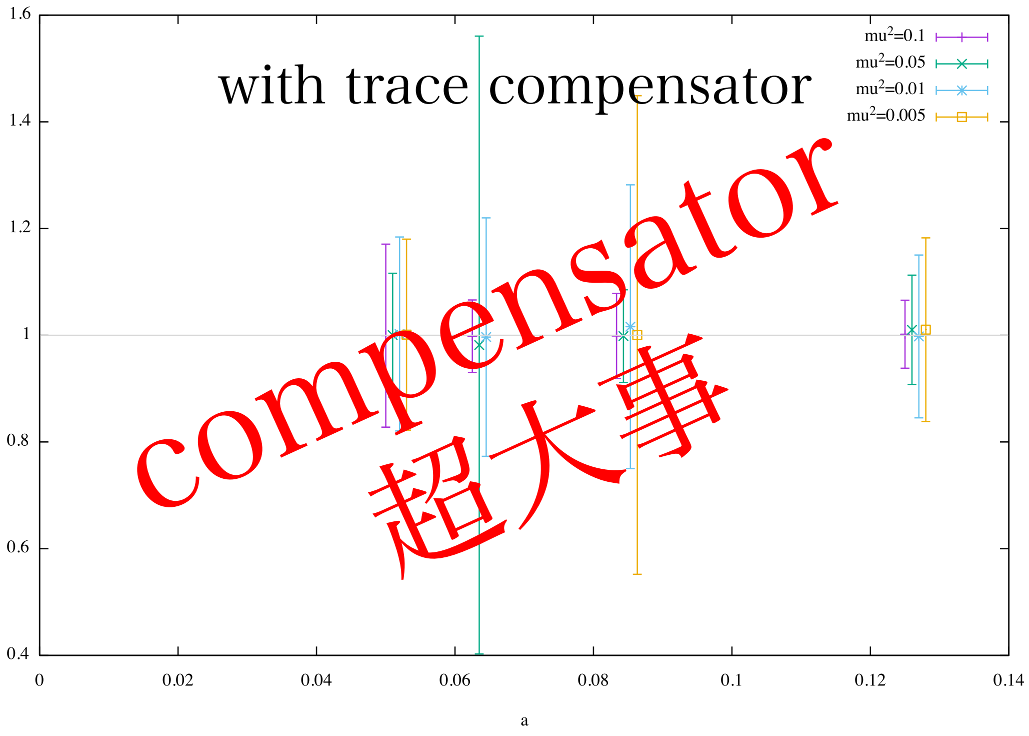
Trivial WT identity for Q-symmetry

basic relation (for $\mu = 0$)

$$0 = \langle Q\Xi \rangle = \langle S_b + S_f \rangle = \langle S_b \rangle - \frac{\#(\text{fermions})}{2}$$

$$\Rightarrow \quad \langle \mathcal{A} S_b \rangle + \frac{\mu^2}{2} \langle \mathcal{A}\Xi \sum_s \mathrm{Tr}(\Phi_s \eta_s) \rangle = \frac{N_c^2 - 1}{2}(N_S + N_L)\langle \mathcal{A} \rangle$$





Identity 1 from $U(1)_V$ symmetry

Site action

$$\frac{1}{2g_0^2}\sum_s \alpha_s Q\,\mathrm{Tr}\left(\frac{1}{4}\eta_s[\phi_s,\bar{\phi}_s]\right)=\frac{1}{2g_0^2}\sum_s \alpha_s\,\mathrm{Tr}\left(\frac{1}{4}[\phi_s,\bar{\phi}_s]^2\right)+\frac{1}{2g_0^2}\sum_s \alpha_s\,\mathrm{Tr}\left(-\frac{1}{4}\eta_s[\phi_s,\eta_s]\right)$$

Face action

Q-exact

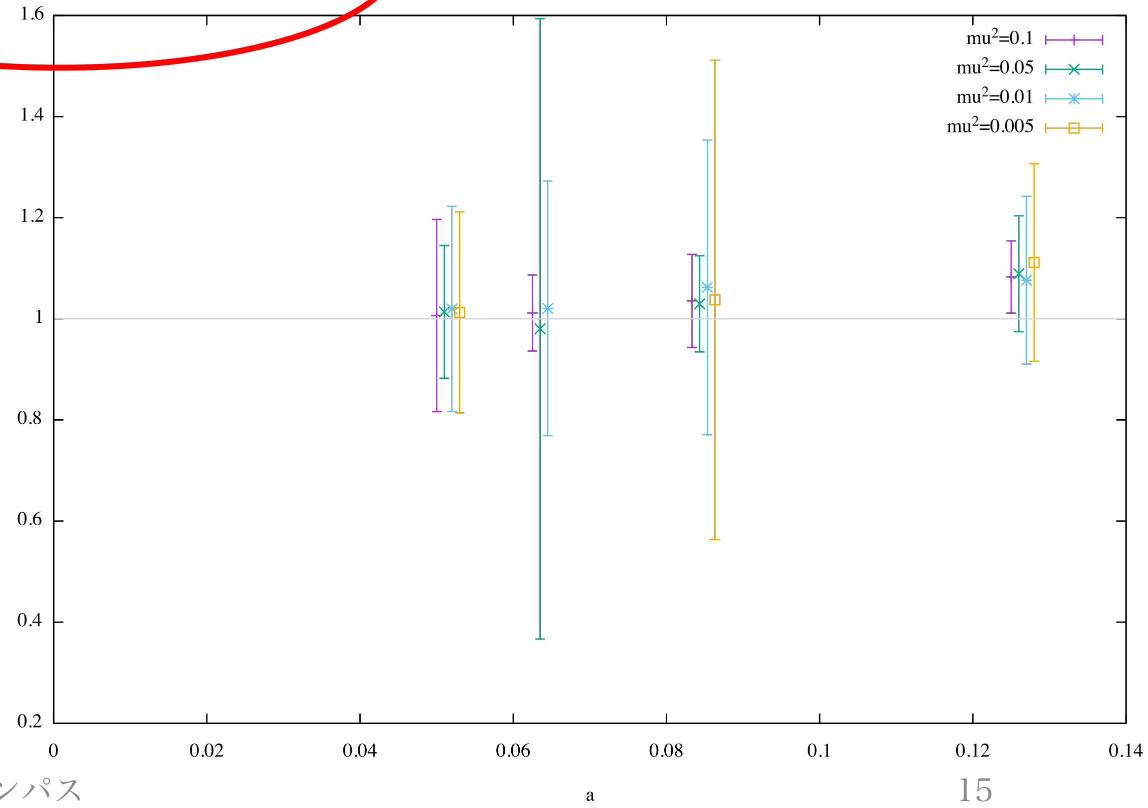
$U(1)_V$ doublet

$$\frac{1}{2g_0^2}\sum_f \alpha_f Q\,\mathrm{Tr}\left(\chi_f Y_f\right)=\frac{1}{2g_0^2}\sum_f \alpha_f\,\mathrm{Tr}\left(Y_f^2\right)+\frac{1}{2g_0^2}\sum_f \alpha_f\,\mathrm{Tr}\left(-\chi_f[\phi_f,\chi_f]\right)$$

$$\Downarrow$$

$$\left\langle\frac{1}{2g_0^2}\sum_s \alpha_s\,\mathrm{Tr}\left(\frac{1}{4}[\phi_s,\bar{\phi}_s]^2\right)\right\rangle+\left\langle\frac{1}{2g_0^2}\sum_f \frac{\alpha_f\beta_f^2}{4}\mathrm{Tr}(\Omega(U_f)^2)\right\rangle$$

$$\rightarrow\frac{1}{2}N_F(N_c^2-1)\quad(a\rightarrow 0)$$



Identity 2 from $U(1)_V$ symmetry

$$\frac{1}{2g_0^2}\sum_l \alpha_l Q\mathrm{Tr}\left(-i\lambda_l(U_l\bar{\phi}_{t(l)}U_l^{-1}-\bar{\phi}_{s(l)})\right)=\frac{1}{2g_0^2}\sum_l \alpha_l \mathrm{Tr}\left(|D_l\phi|^2+i\lambda_l D_l\eta-\lambda_l\lambda_l(U_l\bar{\phi}_{t(l)}U_l^{-1}+\bar{\phi}_{s(l)})\right)$$

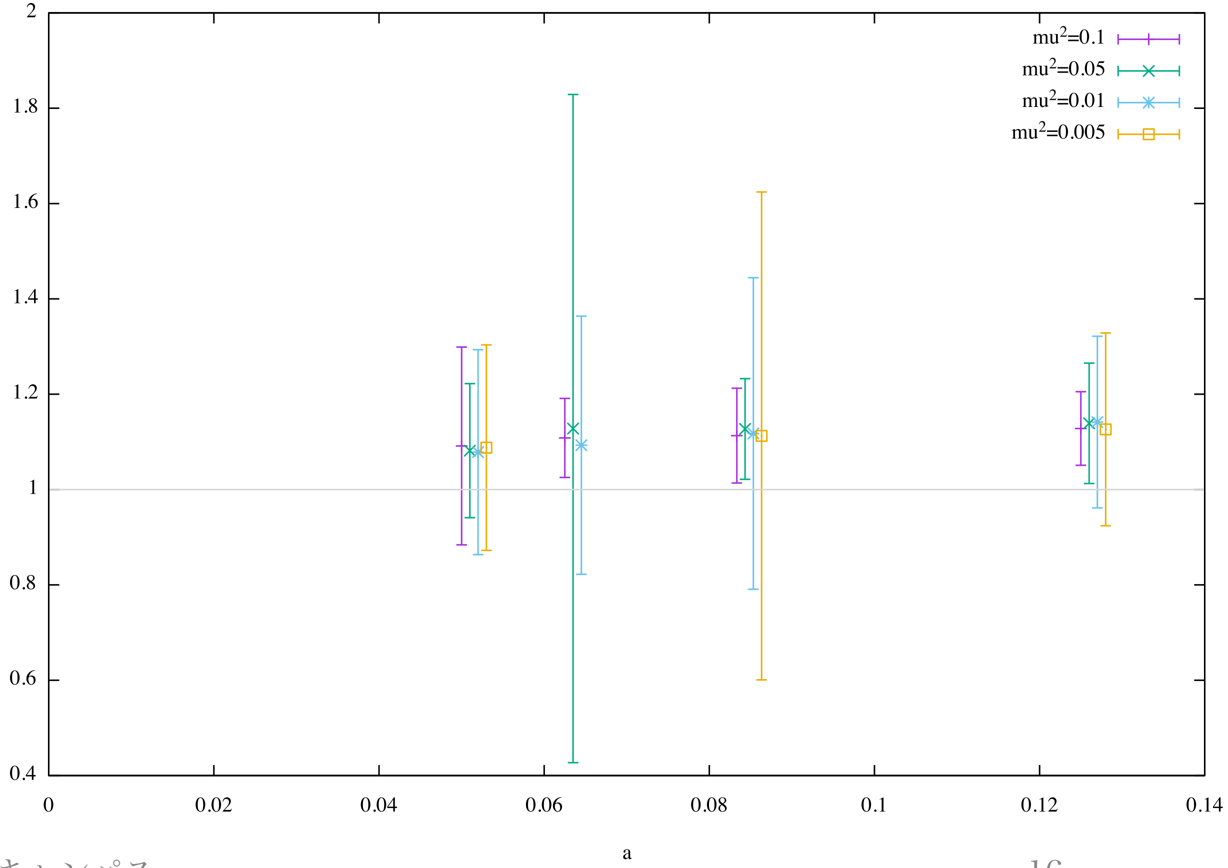
Q-exact

$$\frac{1}{2g_0^2}\sum_f \alpha_f Q\mathrm{Tr}\left(-i\beta_f\chi_f\Omega(U_f)\right)=\frac{1}{2g_0^2}\sum_f \alpha_f\beta_f \mathrm{Tr}\left(-iY_f\Omega(U_f)+i\chi_f Q\Omega(U_f)\right)$$

$U(1)_V$ doublet

$$2\left\langle S_B^F\right\rangle+\left\langle\frac{1}{2g_0^2}\sum_l \alpha_l \mathrm{Tr}\left(\lambda_l\lambda_l(U_l\bar{\phi}_{t(l)}U_l^{-1}+\bar{\phi}_{s(l)})\right)\right\rangle$$

$$\to \left\langle S_B^L\right\rangle \quad (a\to 0)$$



We should check $U(1)_V$ WT identity

In the continuum theory

$$J_V^\mu = \mathrm{Tr}\left\{\frac{1}{2}E^{\mu\nu}\lambda_\nu\eta - \lambda^\mu\chi\right\}$$

For a $U(1)_V$ invariant operator $\mathcal{O}(x)$,

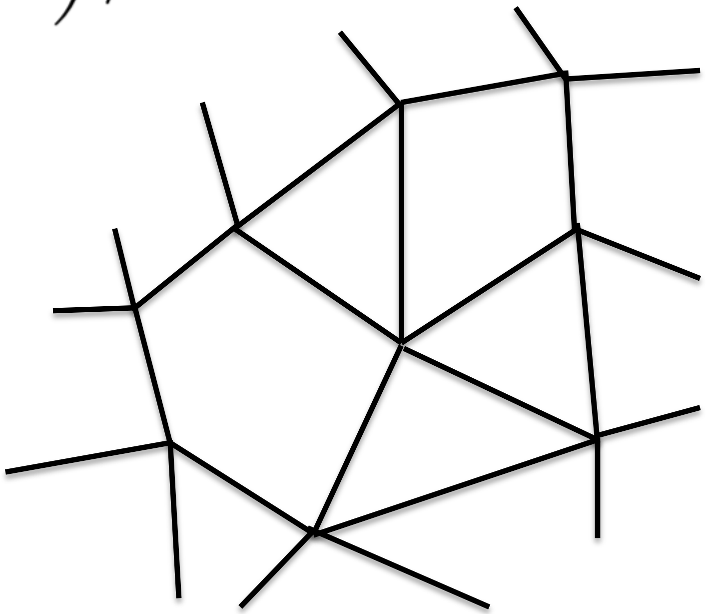
$$\left\langle \mathcal{O}(y)\, \mathcal{D}^\mu J_\mu^V(x) \right\rangle = \left\langle \mathcal{O}(y) \left(\frac{1}{2}\nabla \times \mathrm{Tr}(\lambda\eta) - \nabla \cdot \mathrm{Tr}(\lambda\chi)\right)\right\rangle = 0$$

problem 1

There are infinitely many ways to construct composite operators on the lattice.

problem 2

How to define Rotation and Divergence on lattice?



Hint: exact $U(1)_R$ current on the lattice

cf) Ohta-san's talk

continuum

$$\delta S_{cont} = \frac{1}{2g^2}\int d^2x\sqrt{g}(2i\partial_\mu\theta)\mathrm{Tr}\left\{2i(\phi\mathcal{D}^\mu\bar{\phi}-\bar{\phi}\mathcal{D}^\mu\phi)+2E^{\mu\nu}\lambda_\nu\chi+\lambda^\mu\eta\right\}$$

lattice ($\theta_s \equiv \theta\delta_{s,s_0}$)

$$\delta S_{lat} = \frac{2i\theta}{2g_0^2}\left(\sum_{l\in<\cdot,s_0>}-\sum_{l\in<s_0,\cdot>}\right)\alpha_l\mathrm{Tr}\left(\Phi_{s(l)}U_l\bar{\Phi}_{t(l)}U_l^{-1}-\bar{\Phi}_{s(l)}U_l\Phi_{t(l)}U_l^{-1}\right)$$

DIVERGENCE

$$+\frac{2i\theta}{2g_0^2}\left(\sum_{l\in<\cdot,s_0>}-\sum_{l\in<s_0,\cdot>}\right)\alpha_l\mathrm{Tr}\left(\frac{i}{2}\lambda_lU_l\eta_{t(l)}U_l^{-1}+\lambda_l\lambda_lU_l\bar{\Phi}_{t(l)}U_l^{-1}\right)$$

DIVERGENCE

$$+\frac{2i\theta}{2g_0^2}\left(\sum_{l\in<s_0,\cdot>}\sum_{f\in F_l}-\sum_{f=s_0}\sum_{l\in L_f}\right)(i\alpha_f\beta_f\epsilon_{f,l})\left\{\frac{1}{2B(U_f)}\mathrm{Tr}\left(\chi_fX_{f,l}\lambda_lY_{f,l}+\chi_fY_{f,l}^\dagger\lambda_lX_{f,l}^\dagger\right)-\frac{1}{\epsilon^2}\frac{1}{2B(U_f)^2}\mathrm{Tr}\left(\chi_f(U_f-U_f^{-1})\right)\mathrm{Tr}\left(X_{f,l}\lambda_lY_{f,l}-Y_{f,l}^\dagger\lambda_lX_{f,l}^\dagger\right)\right\}$$

ROTATION

$$B(U_f)=1-\frac{1}{\epsilon^2}\mathrm{Tr}\left(2-U_f-U_f^{-1}\right).$$

This correspondence will be a guiding principle to construct "$U(1)_V$ current", rotation and divergence on the lattice.

on going…

Summary and Future Works

- We explicitly discretize 2-sphere.
- Q-symmetry is preserved as expected.
- The relations expected from the $U(1)_V$ symmetry seems to hold in the continuum limit.
- $U(1)_V$ WT identity, restoration of S^2 geometry   Kamata-Ohta-Misumi-S.M. to be appeared
- Other discretization (fullerene-like discretization?)
- N=(4,4) and N=(8,8) theory
- Simulation of 4D N=4 SYM by hybrid method   cf) Hanada-Sugino-S.M. 2012
- Theory with matters
- Connection to Dynamical triangulation? (supersymmetric dynamical triangulation?)