量子計算による
場と時空のダイナミクス

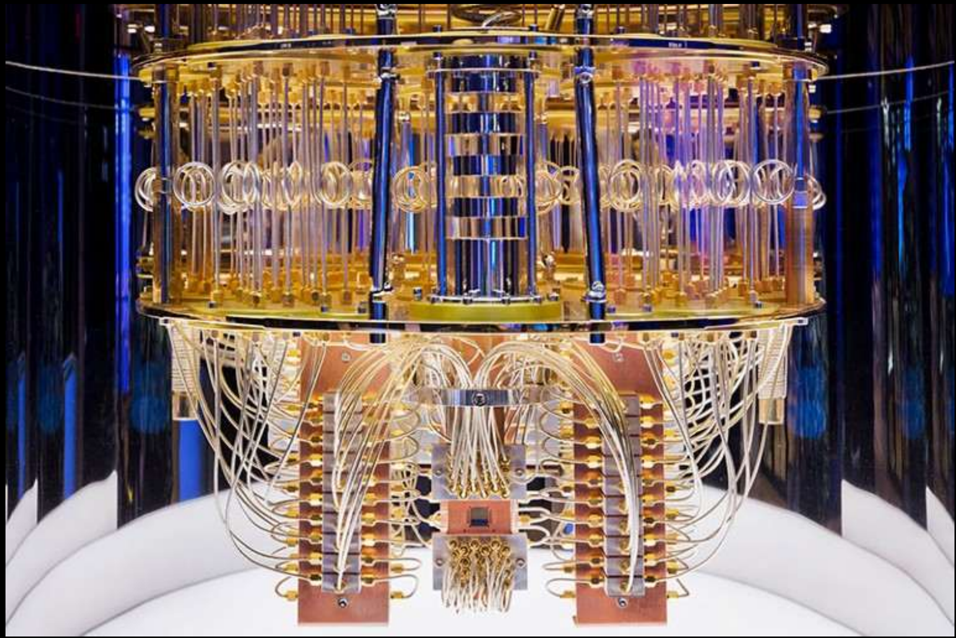
Masazumi Honda

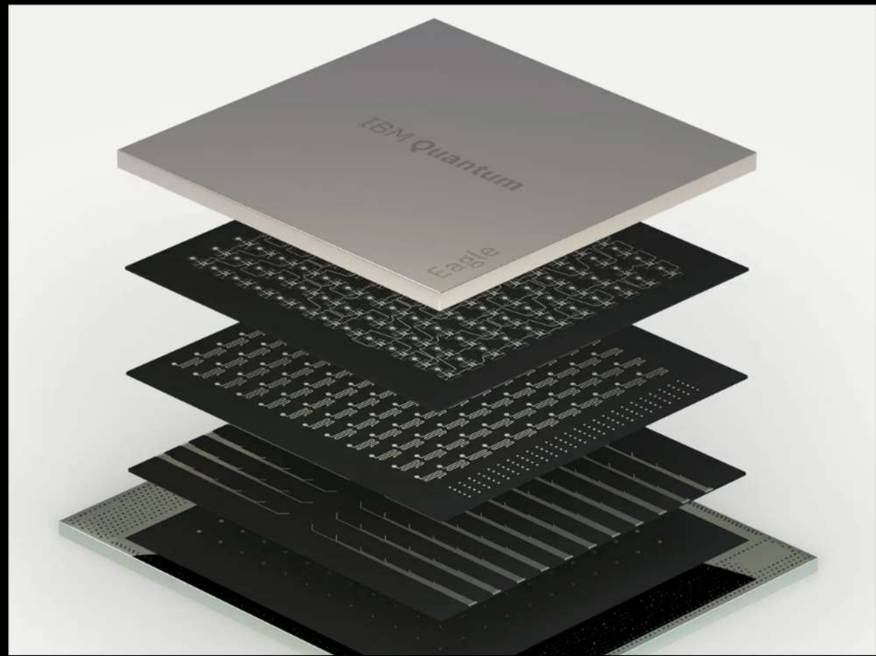
(本多正純)

11th Sep, 2025

離散的手法による場と時空のダイナミクス2025
@明治学院大学

Quantum computer sounds growing well…





Article

Evidence for the utility of quantum computing before fault tolerance

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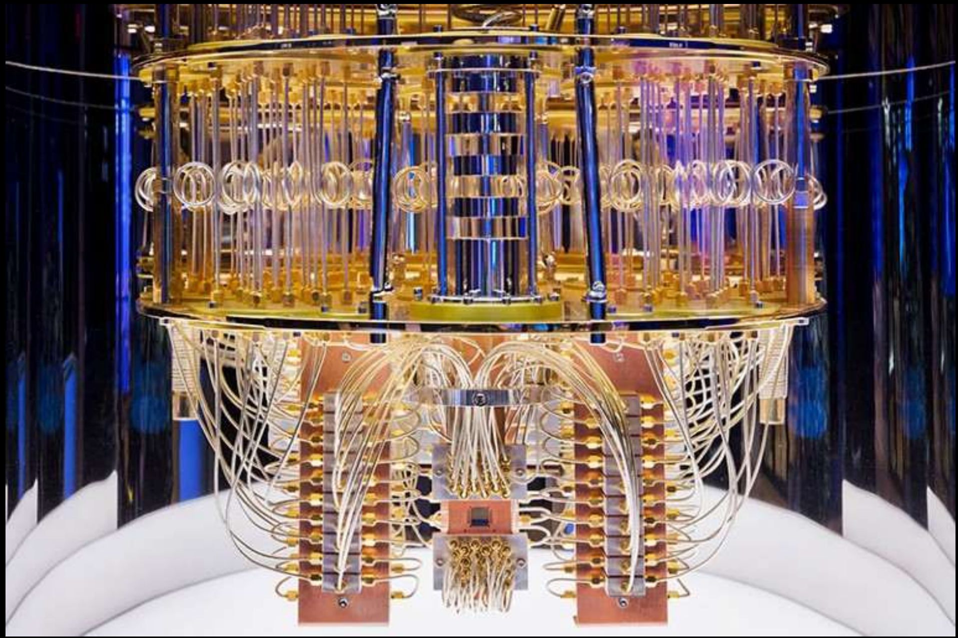
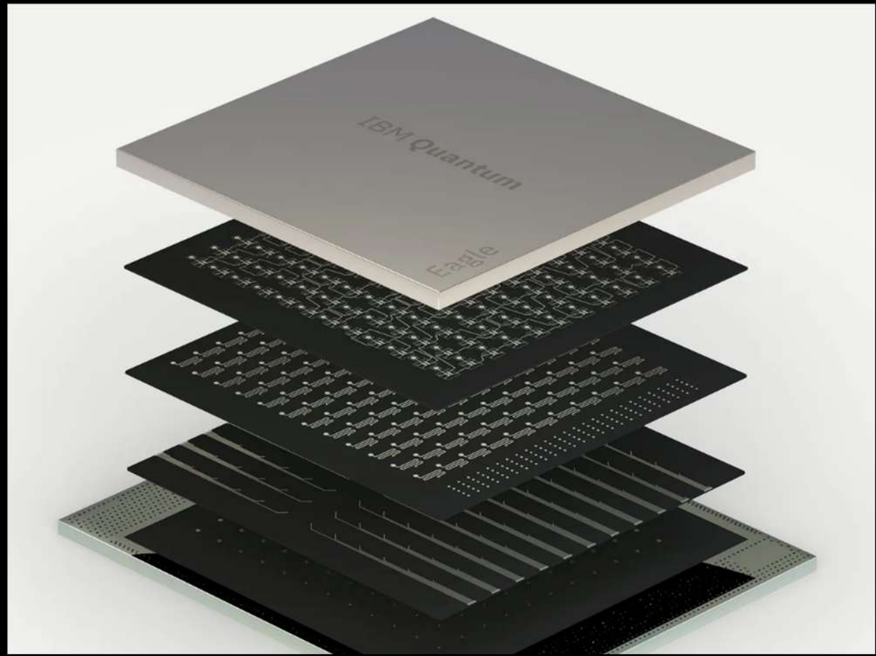
Youngseok Kim[1,6]✉, Andrew Eddins[2,6]✉, Sajant Anand[3], Ken Xuan Wei[1], Ewout van den Berg[1], Sami Rosenblatt[1], Hasan Nayfeh[1], Yantao Wu[3,4], Michael Zaletel[3,5], Kristan Temme[1] & Abhinav Kandala[1]✉

Quantum computing promises to offer substantial speed-ups over its classical

Quantum computer sounds growing well…



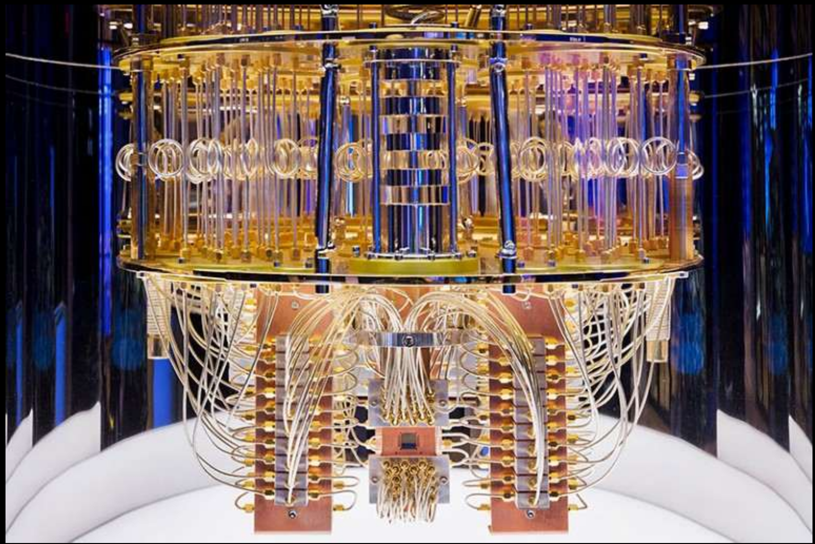


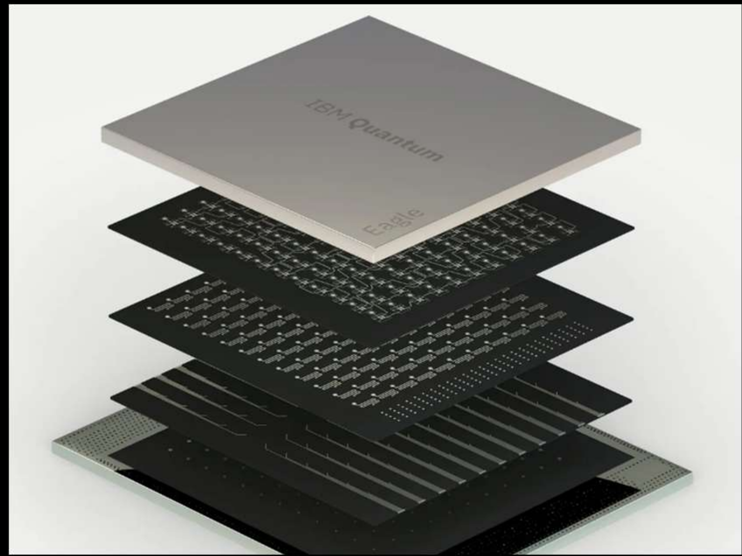
Article

Evidence for the utility of quantum computing before fault tolerance

How can we use it for us?

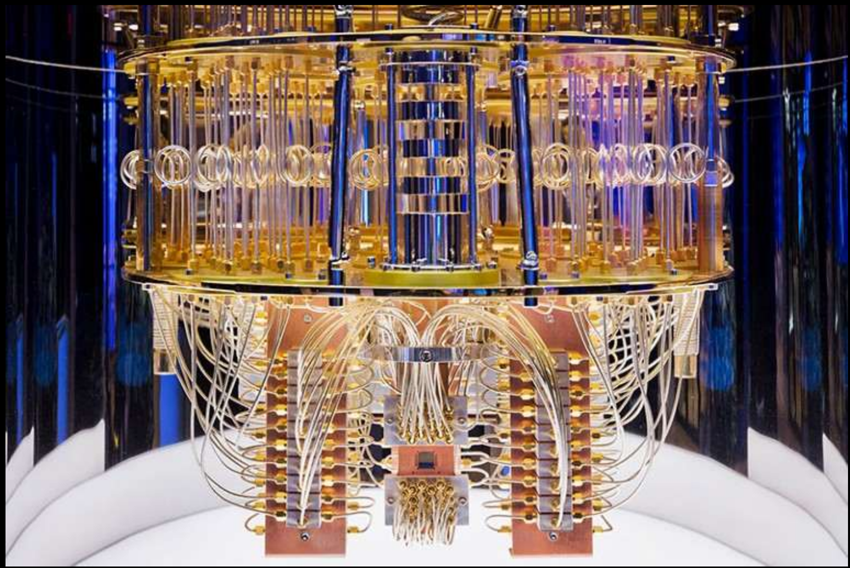
Applications mentioned in media ?

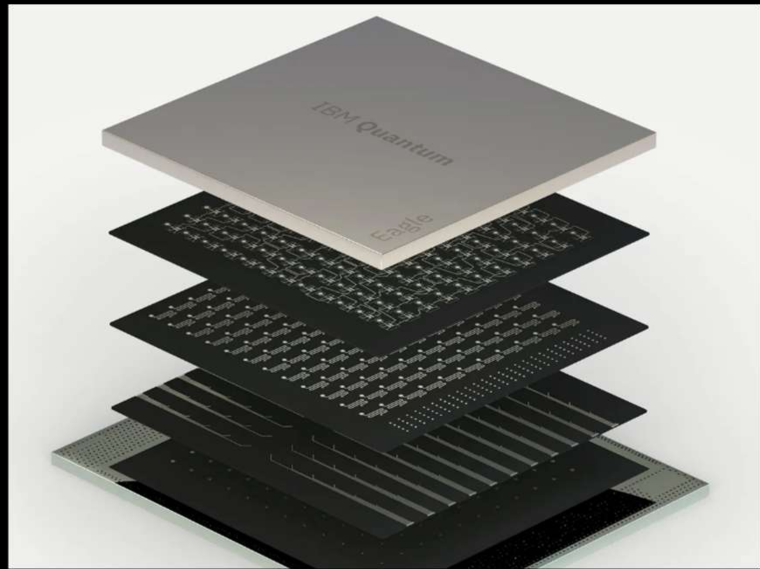


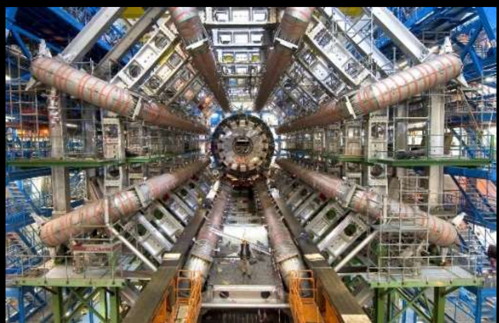


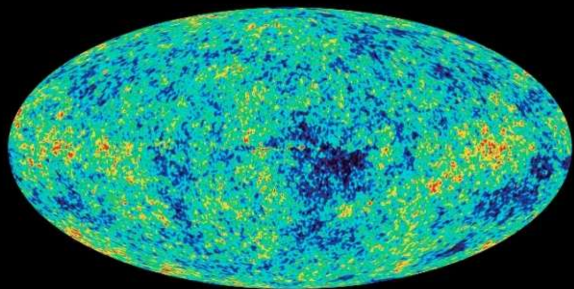
etc...

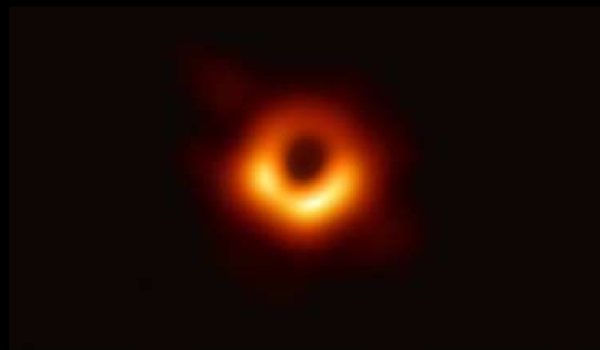
In my mind…











etc…

What is meant by

“Application of Quantum Computation to High Energy Physics” ??

In general, it is

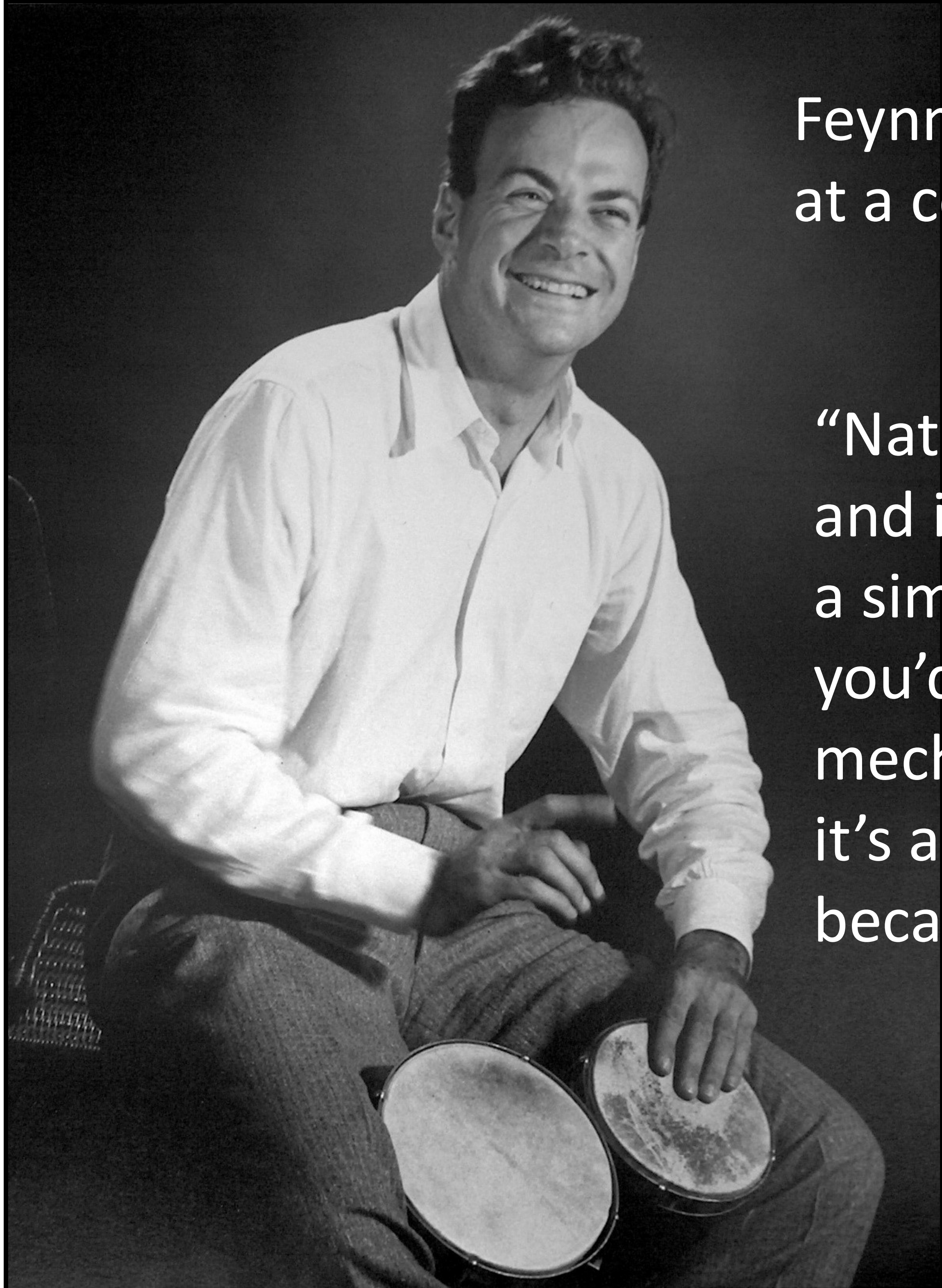
to replace (a part of) computations by quantum algorithm

Therefore,

physical meaning of qubits in quantum computer depends on contexts

Here,

qubits = states in quantum system



Feynman as a keynote speaker
at a conference in MIT (1981):

“Nature isn’t classical, dammit,
and if you want to make
a simulation of Nature,
you’d better make it quantum
mechanical, and by golly
it’s a wonderful problem
because it doesn’t look so easy.”

Focus of this talk:

Application of Quantum Computation
to
Quantum Field Theory & Gravity

- Generic motivation:

simply would like to use powerful computers?

- Specific motivation:

Quantum computation is suitable for operator formalism

$\longrightarrow$ Liberation from infamous sign problem in Monte Carlo?

(skipped)

Cost of operator formalism

We have to play with huge vector space

since QFT typically has ∞-dim. Hilbert space

regularization needed!

Technically, computers have to

memorize huge vector & multiply huge matrices

Cost of operator formalism

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Quantum computers do this job?

Contents

Qubit = Quantum Bit

Qubit = Quantum system w/ 2 dim. Hilbert space

Basis:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

"computational basis"

Generic state:

$$\alpha|0\rangle + \beta|1\rangle \qquad \text{w/} \quad |\alpha|^2 + |\beta|^2 = 1$$

Ex.) Spin 1/2 system:

$$|0\rangle = |\uparrow\rangle, \qquad |1\rangle = |\downarrow\rangle$$

(We don't need to mind how it is realized as "users")

Multiple qubits

2 qubits – 4 dim. Hilbert space:

$$|\psi\rangle = \sum_{i,j=0,1} c_{ij}|ij\rangle, \qquad |ij\rangle \equiv |i\rangle \otimes |j\rangle$$

$$|00\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \quad |01\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \quad |10\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \quad |11\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

N qubits – 2^N dim. Hilbert space:

$$|\psi\rangle = \sum_{i_1,\cdots i_N=0,1} c_{i_1\cdots i_N}|i_1\cdots i_N\rangle,$$

$$|i_1 i_2 \cdots i_N\rangle \equiv |i_1\rangle \otimes |i_2\rangle \otimes \cdots \otimes |i_N\rangle$$

Rule of the game

Do something interesting by a combination of

1. action of Unitary operators:

$$|\psi\rangle \quad \text{———} \boxed{U} \text{———} \quad U|\psi\rangle$$

&

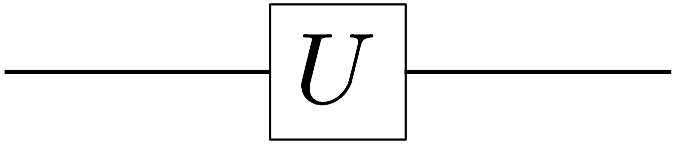
2.

Rule of the game

Do something interesting by a combination of

1. action of Unitary operators:

$|\psi\rangle$

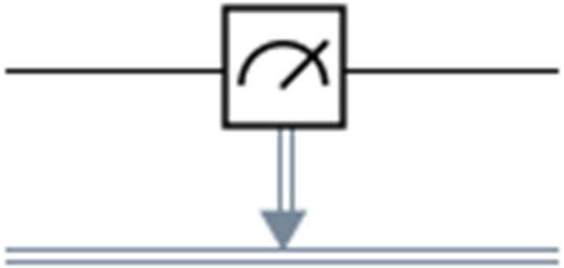


$U|\psi\rangle$

&

2. measurements:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



c (classical number)

$$\begin{cases} c = 0 \text{ w/ probability } |\alpha|^2 \\ c = 1 \text{ w/ probability } |\beta|^2 \end{cases}$$

Unitary gates used here

X, Y, Z gates:

(just Pauli matrices)

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

X is "NOT": $X|0\rangle = |1\rangle, \; X|1\rangle = |0\rangle$

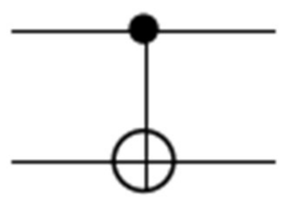
R_X, R_Y, R_Z gates:

$$R_X(\theta) = e^{-\frac{i\theta}{2}X}, \qquad R_Y(\theta) = e^{-\frac{i\theta}{2}Y}, \qquad R_Z(\theta) = e^{-\frac{i\theta}{2}Z}$$

Controlled X (NOT) gate:

$$\begin{cases} CX|00\rangle = |00\rangle, & CX|01\rangle = |01\rangle, \\ CX|10\rangle = |11\rangle, & CX|11\rangle = |10\rangle \end{cases}$$

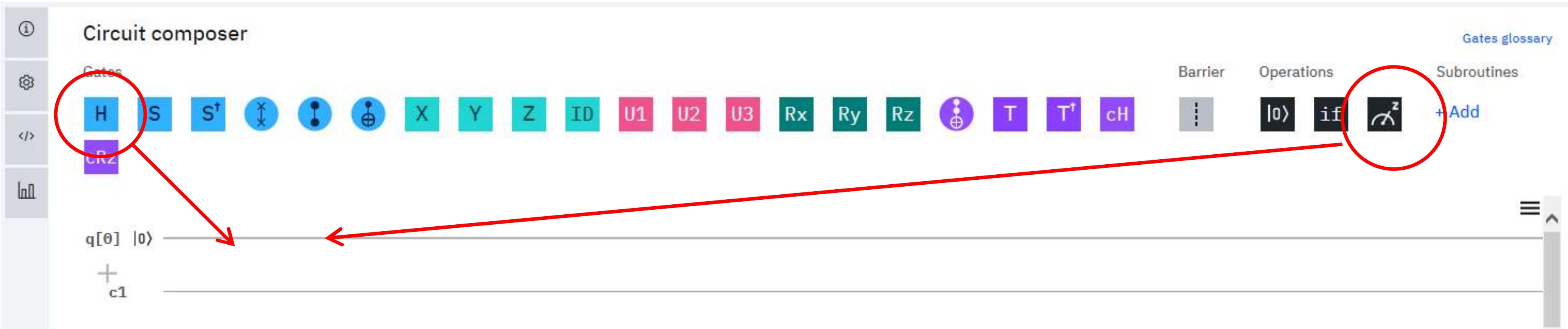
$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} =$$



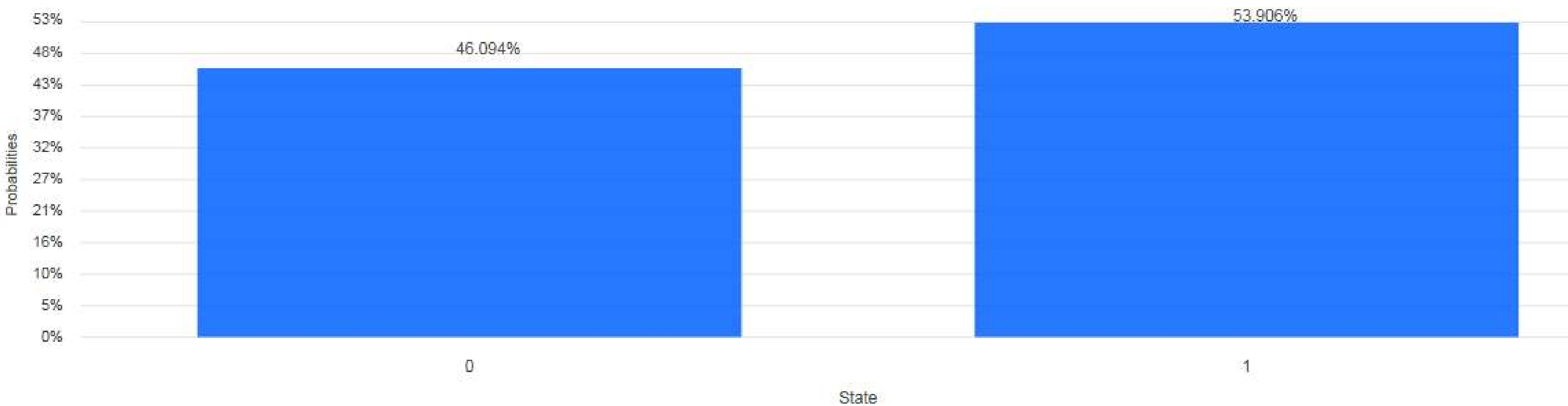
Atmosphere (?) of using quantum computer…

Suppose we'd like to measure the state: $H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Screenshot of IBM Quantum:



Output of 1024 times measurements ("shots") :



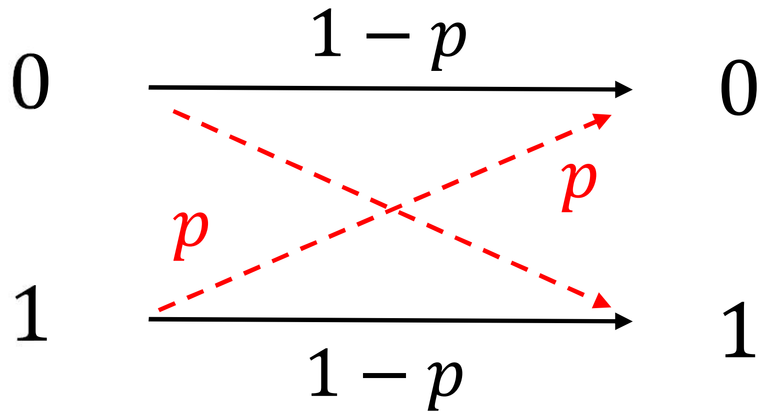
Idea: express physical quantities in terms of "probabilities"
& measure the "probabilities"

Errors in classical computers

Computer interacts w/ environment ⇒ error/noise

Errors in classical computers

Computer interacts w/ environment ⟹ error/noise

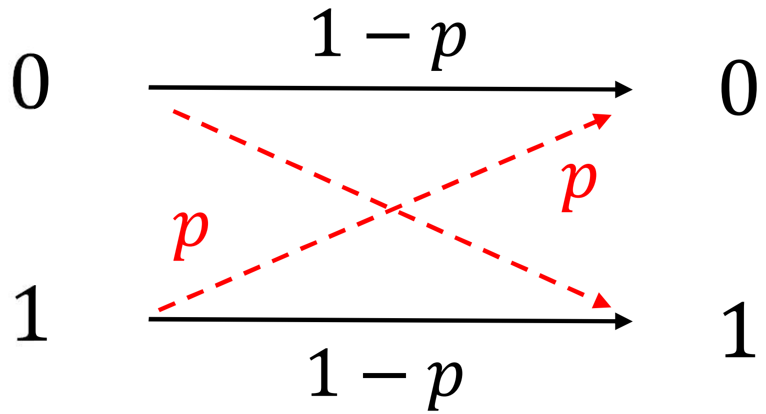


Suppose we send a bit but have "error" in probability p

A simple way to correct errors:

Errors in classical computers

Computer interacts w/ environment $\Rightarrow$ error/noise



Suppose we send a bit but have "error" in probability p

A simple way to correct errors:

① Duplicate the bit (encoding): $0 \rightarrow 000, \quad 1 \rightarrow 111$

② Error detection & correction by "majority voting":

$$001 \rightarrow 000, \quad 011 \rightarrow 111, \quad \text{etc...}$$

$\Rightarrow$ $P_{\text{failed}} = 3p^2(1-p) + p^3$ (improved if $p < 1/2$)

Errors in quantum computers

Computer interacts w/ environment ⇨ error/noise

Unknown unitary operators are multiplied:

(in addition to decoherence & measurement errors)

$$|\psi\rangle \xrightarrow{\text{error!}} U|\psi\rangle$$

not only bit flip!

Errors in quantum computers

Computer interacts w/ environment ⇒ error/noise

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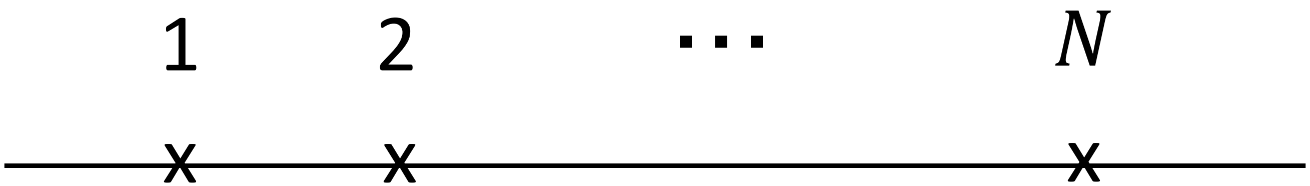
not only bit flip!

We need to include "quantum error corrections"
but it seems to require a huge number of qubits

$\sim$ major obstruction of the development

Contents

The (1+1)d transverse Ising model



Hamiltonian (w/ open b.c.):

(X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

$$\hat{H} = -J \sum_{n=1}^{N-1} Z_n Z_{n+1} - h \sum_{n=1}^{N} X_n$$

Let's construct the time evolution op. $e^{-i\hat{H}t}$

Time evolution operator

Time evolution of any state is studied by acting the operator

$$e^{-i\hat{H}t} = e^{-i(H_X + H_{ZZ})t}$$

where

$$H_X = -h(X_1 + X_2), \quad H_{ZZ} = -JZ_1Z_2$$

How do we express this in terms of elementary gates?

(such as $X, Y, Z, R_{X,Y,Z}, CX$ etc…)

Step 1: Suzuki-Trotter decomposition:

Time evolution operator

Time evolution of any state is studied by acting the operator

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How do we express this in terms of elementary gates?

(such as $X, Y, Z, R_{X,Y,Z}, CX$ etc...)

Step 1: Suzuki-Trotter decomposition:

(∃higher order improvements)

$$e^{-i\widehat{H}t} = \left(e^{-i\widehat{H}\frac{t}{M}}\right)^M$$

(M: large positive integer)

$$\simeq \left(e^{-iH_X\frac{t}{M}} e^{-iH_{ZZ}\frac{t}{M}}\right)^M + \mathcal{O}(1/M)$$

Time evolution operator (Cont'd)

$$e^{-i\hat{H}t} \simeq \left(e^{-iH_X \frac{t}{M}} e^{-iH_{ZZ}\frac{t}{M}}\right)^M$$

The 1st one is trivial:

acting on qubit 2 acting on qubit 1

$$e^{-iH_X\frac{t}{M}} = e^{-i\frac{ht}{M}X_2} e^{-i\frac{ht}{M}X_1} = R_X^{(2)}\left(\frac{2ht}{M}\right) R_X^{(1)}\left(\frac{2ht}{M}\right)$$

The 2nd one is nontrivial:

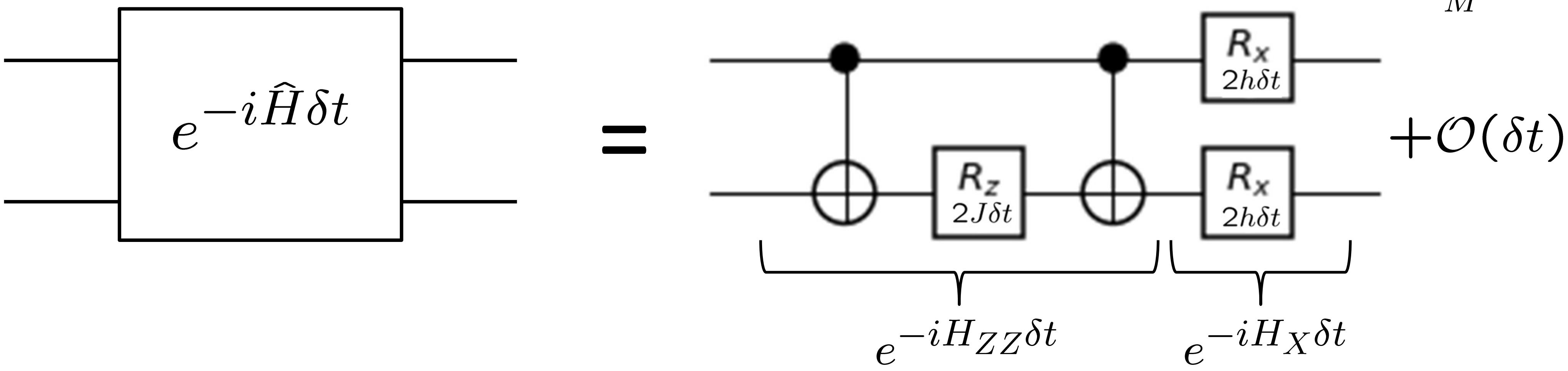
$$e^{-iH_{ZZ}\frac{t}{M}} = e^{-i\frac{Jt}{M}Z_1Z_2} = \cos\frac{Jt}{M} - iZ_1Z_2\sin\frac{Jt}{M}$$

One can show

$$e^{-i\frac{Jt}{M}Z_1Z_2} = CXR_Z^{(2)}\left(\frac{2Jt}{M}\right)CX$$

“Computational cost” for large size system

$$\delta t = \frac{t}{M} \ll 1$$



Classical computer

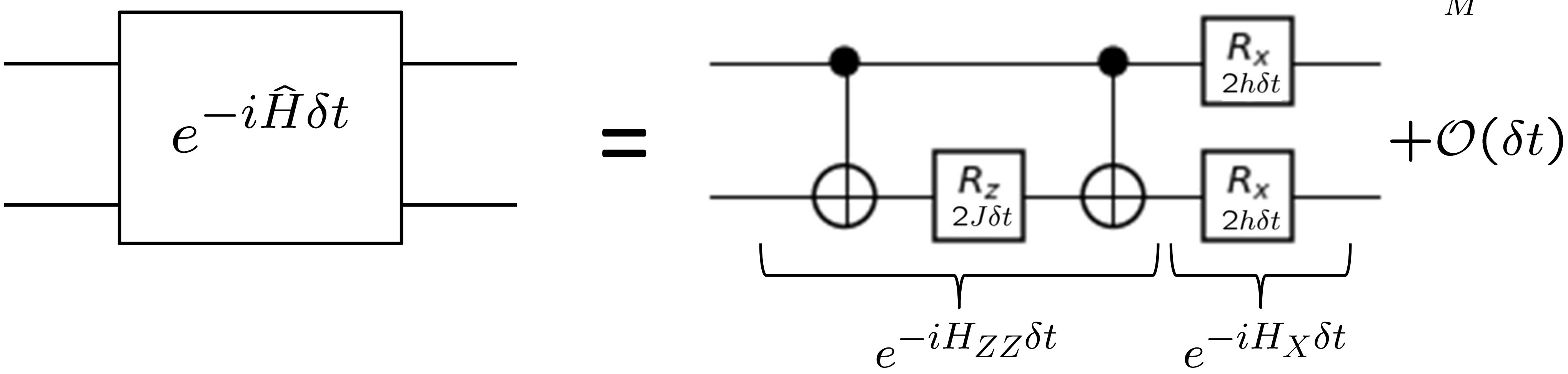
multiplications of matrices to vectors w/ sizes $= 2^N$

exponentially large steps

Quantum computer

"Computational cost" for large size system

$$\delta t = \frac{t}{M} \ll 1$$



Classical computer

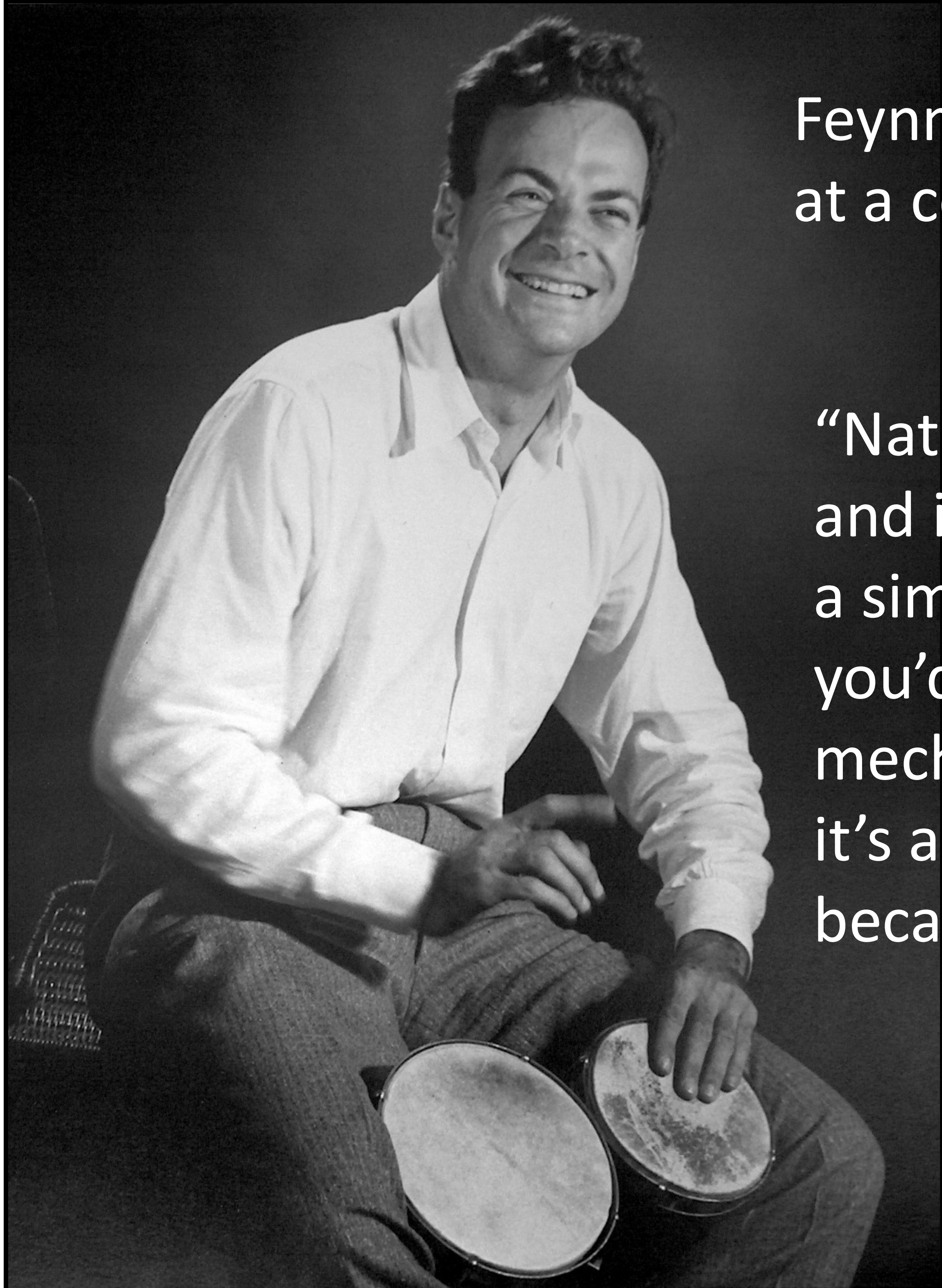
multiplications of matrices to vectors w/ sizes $= 2^N$

exponentially large steps

Quantum computer

- time evolution $= \mathcal{O}(NM)$ experimental operations

polynomial steps



Feynman as a keynote speaker at a conference in MIT (1981):

"Nature isn't classical, dammit, and if you want to make a simulation of Nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem because it doesn't look so easy."

Contents

“Regularization” of Hilbert space

Hilbert space of QFT is typically ∞ dimensional

$\longrightarrow$ Make it finite dimensional!

- Fermion is easiest (up to doubling problem)

  —— Putting on spatial lattice, Hilbert sp. is finite dimensional

- scalar

  —— Hilbert sp. at each site is ∞ dimensional

  (need truncation or additional regularization)

- gauge field (w/ kinetic term)

  —— no physical d.o.f. in 0+1D/1+1D (w/ open bdy. condition)

  —— ∞ dimensional Hilbert sp. in higher dimensions

Citation history of "Hamiltonian Formulation of Wilson's Lattice Gauge Theories" by Kogut-Susskind

(totally 2565 at this moment)

(1+1)d free Dirac fermion

Continuum:

$$\psi(x) = \begin{pmatrix} \psi_u(x) \\ \psi_d(x) \end{pmatrix} \qquad \gamma^0 = \sigma_3, \quad \gamma^1 = i\sigma_2$$

$$H = \int dx\left[-i\bar{\psi}\gamma^1\partial_1\psi + m\bar{\psi}\psi\right]$$

$$= \int dx\left[-i(\psi_u^\dagger\partial_1\psi_d + \psi_d^\dagger\partial_1\psi_u) + m(\psi_u^\dagger\psi_u - \psi_d^\dagger\psi_d)\right]$$

$$\Downarrow$$

Lattice (w/ N sites and spacing a):

"Staggered fermion" [Susskind, Kogut-Susskind '75]

$$\frac{\chi_n}{a^{1/2}} \longleftrightarrow \psi(x) = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} \begin{matrix} \rightarrow \text{odd site} \\ \rightarrow \text{even site} \end{matrix}$$

$$H = -\frac{i}{2a}\sum_{n=1}^{N-1}\left(\chi_n^\dagger\chi_{n+1} - \chi_{n+1}^\dagger\chi_n\right) + m\sum_{n=1}^{N}(-1)^n\,\chi_n^\dagger\chi_n$$

$$\{\chi_m, \chi_n^\dagger\} = \delta_{\mathrm{mn}}, \ \{\chi_m, \chi_n\} = 0$$

Jordan-Wigner transformation

$$\{\chi_m, \chi_n^\dagger\} = \delta_{\mathrm{mn}}, \ \{\chi_m, \chi_n\} = 0$$

This is satisfied by the operator: [Jordan-Wigner'28]

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2} \left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i \right)$$

(X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

Jordan-Wigner transformation

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(X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

Then the system is mapped to the spin system:

$$\boxed{\hat{H} = \frac{w}{2}\sum_{n=1}^{N-1}\left(X_n X_{n+1} + Y_n Y_{n+1}\right) + \frac{m}{2}\sum_{n=1}^{N}(-1)^n Z_n}$$

Now we can apply quantum algorithms to QFT!

Scalar field theory

Continuum Hamiltonian:

$$H = \int d^d\boldsymbol{x} \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\partial_i\phi)^2 + V(\phi)\right]$$

$$\int d^d x \to a^d \sum_n ,$$

$$\partial_\mu\phi(x) \to \Delta_\mu\phi(x_n) \equiv \frac{\phi(x_n + a e_\mu) - \phi(x_n)}{a}$$

Lattice Hamiltonian (simplest):

$$H = a^d \sum_n \left[\frac{1}{2}\Pi_{\mathrm{n}}^2 + \frac{1}{2}\sum_i (\Delta_i\phi_n)^2 + V(\phi_n)\right]$$

$$[\phi(\boldsymbol{x_m}), \Pi(\boldsymbol{x_n})] = i\delta_{\boldsymbol{m,n}}$$

technically the same as multi-particle QM

Regularization for single particle QM

$$\hat{H} = \frac{1}{2}\hat{p}^2 + \frac{\omega^2}{2}\hat{x}^2 + V_{\text{int}}(\hat{x})$$

Most naïve approach = truncation in harmonic osc. basis:

$$\hat{a} = \sqrt{\frac{\omega}{2}}\,\hat{x} + \frac{i}{\sqrt{2\omega}}\,\hat{p} \;=\; \sum_{n=0}^{\infty} \sqrt{n+1}\,|n\rangle\langle n+1|$$

$$\xRightarrow[\textit{regularize!}]{} \sum_{n=0}^{\Lambda-2} \sqrt{n+1}\,|n\rangle\langle n+1|$$

Then replace $\hat{p}$ & $\hat{x}$ by

$$\hat{x}\,\Big|_{\text{regularized}} \equiv \frac{1}{\sqrt{2\omega}}(\hat{a} + \hat{a}^\dagger)\,\Big|_{\text{regularized}}$$

$$\hat{p}\,\Big|_{\text{regularized}} \equiv \frac{1}{i}\sqrt{\frac{\omega}{2}}\,(\hat{a} - \hat{a}^\dagger)\,\Big|_{\text{regularized}}$$

Regularization for single particle QM (Cont'd)

$$\hat{a}\Big|_{\mathrm{regularized}} = \sum_{n=0}^{\Lambda-2} \sqrt{n+1}\,|n\rangle\langle n+1|$$

We can rewrite the Fock basis in terms of qubits:

$$|n\rangle = |b_{K-1}\rangle|b_{K-2}\rangle\cdots|b_0\rangle \qquad K \equiv \log_2\Lambda$$

$$n = \mathrm{b}_{\mathrm{K}-1}2^{\mathrm{K}-1} + \mathrm{b}_{\mathrm{K}-2}2^{\mathrm{K}-2} + \cdots + b_0 2^0$$ (binary representation)

Regularization for single particle QM (Cont'd)

$$\hat{a}\Big|_{\text{regularized}} = \sum_{n=0}^{\Lambda-2} \sqrt{n+1}\,|n\rangle\langle n+1|$$

We can rewrite the Fock basis in terms of qubits:

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$$n = \mathrm{b}_{\mathrm{K}-1}2^{\mathrm{K}-1} + \mathrm{b}_{\mathrm{K}-2}2^{\mathrm{K}-2} + \cdots + b_0 2^0$$ (binary representation)

Then,

$$|n\rangle\langle n+1| = \otimes_{\ell=0}^{K-1} \underbrace{(|b'_\ell\rangle\langle b_\ell|)}_{\text{either one of}}$$

$$\left(\begin{array}{ll} |0\rangle\langle 0| = \dfrac{1_2 - \sigma_z}{2}, & |1\rangle\langle 1| = \dfrac{1_2 + \sigma_z}{2}, \\ |0\rangle\langle 1| = \dfrac{\sigma_x + i\sigma_y}{2}, & |1\rangle\langle 0| = \dfrac{\sigma_x - i\sigma_y}{2} \end{array} \right)$$

Pure Maxwell theory

Continuum:

$$\mathcal{H} = \frac{1}{2}E_i^2 + \frac{1}{2}B_i^2 \qquad \partial_i E^i = 0$$

$$\Downarrow$$

Lattice:

$$\mathcal{H} = \frac{a^d}{2}\sum_{\boldsymbol{n},i} L_{\boldsymbol{n},i}^2 + \mathrm{Re}\sum_{\text{plaquette}}\sum_{i<j}\prod_{P\in\text{plaquette}} U_P$$

$$[U_{\boldsymbol{m},i}, L_{\boldsymbol{n},j}] = i\delta_{ij}\delta_{\boldsymbol{m},\boldsymbol{n}}$$

Gauss law:

$$\sum_i (L_{\boldsymbol{n}+\boldsymbol{e}_i,i} - L_{\boldsymbol{n},i}) = 0$$

Ex. (1+1)d pure Maxwell theory w/ θ

Continuum:

$$\mathcal{L} = \frac{1}{2g^2} F_{01}^2 + \frac{\theta}{2\pi} F_{01} \xrightarrow{\Pi = \frac{1}{g^2}\dot{A} + \frac{\theta}{2\pi}} \mathcal{H} = \frac{1}{2}\left(\Pi - \frac{\theta}{2\pi}\right)^2$$

Lattice:

$$H = \frac{g^2 a}{2} \sum_n \left(L_n + \frac{\theta}{2\pi}\right)^2 \qquad L_n \leftrightarrow -\frac{\Pi(x)}{g}$$

Gauss law:

$$L_{n+1} - L_n = 0$$

- open b.c.

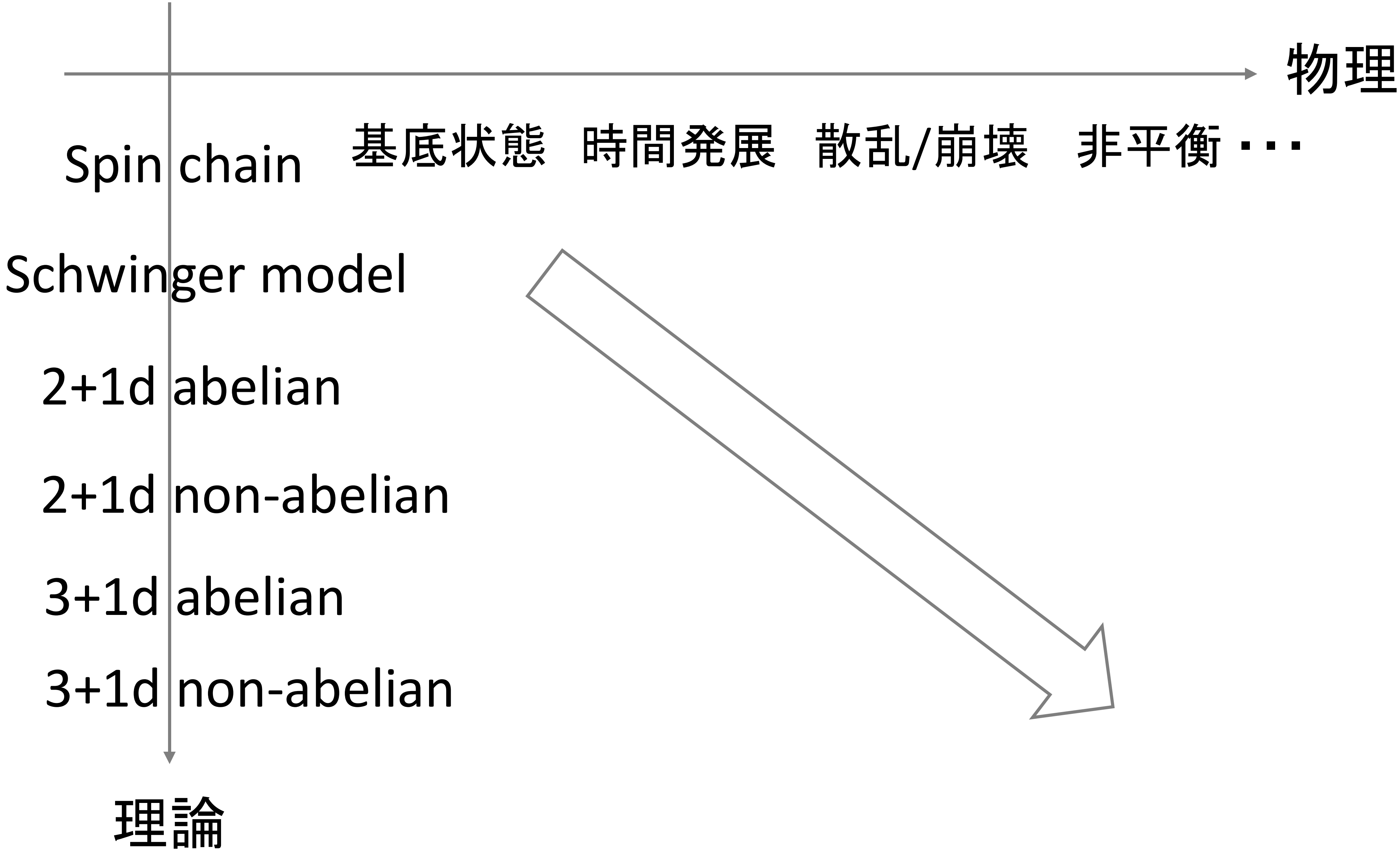
$$L_n = L_{n-1} = L_{n-2} = \cdots = L_1 = (b.c.)$$

- p.b.c.

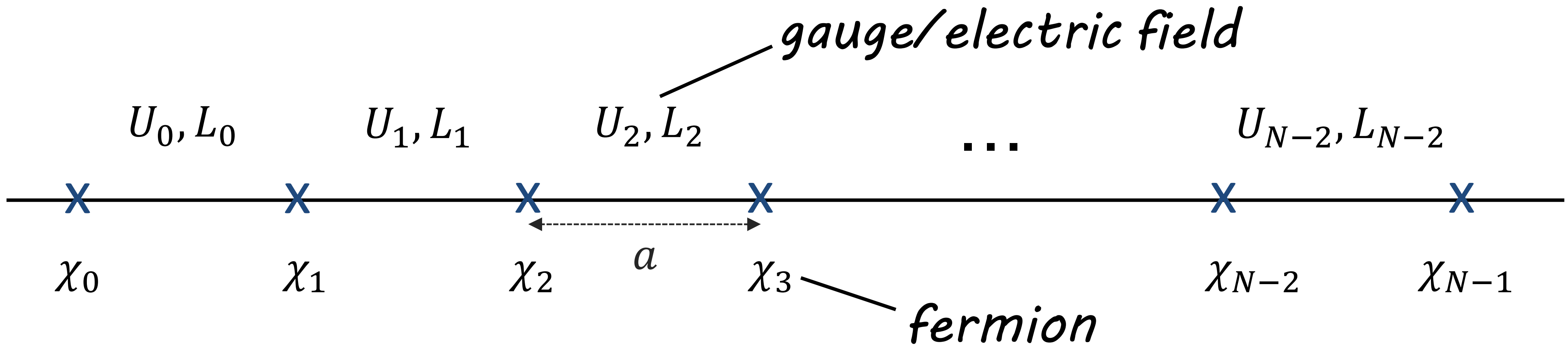
$$L_n = L_{n-1} = \cdots = L_1 = \cdots = L_{n+1} = L_n$$

one d.o.f. remains

分野の大体の研究の流れ (?)



Charge-q Schwinger model



$$H = J \sum_{n=0}^{N-2} \left(L_n + \frac{\theta_0}{2\pi} \right)^2 - iw \sum_{n=0}^{N-2} \left[\chi_n^\dagger (U_n)^q \chi_{n+1} - \text{h.c.} \right] + m \sum_{n=0}^{N-1} (-1)^n \chi_n^\dagger \chi_n$$

$$[L_n, U_m] = U_m \delta_{nm}, \quad \{\chi_n, \chi_m^\dagger\} = \delta_{nm}$$

Physical states are subject to Gauss law:

$$(L_n - L_{n-1})|\text{phys}\rangle \quad = \quad q\left[\chi_n^\dagger \chi_n - \frac{1-(-1)^n}{2}\right] |\text{phys}\rangle$$

"$\nabla \cdot \vec{E}\,(x)$" "$\rho(x)$ "

Schwinger model as qubits

1. Take open b.c. & solve Gauss law:

$$L_n = L_{-1} + q \sum_{j=1}^{n} \left(\chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2} \right) \qquad \text{w/ } L_{-1} = 0$$

2. Take the gauge $U_n = 1$

3. Map to spin system: $\chi_n = \frac{X_n - \mathrm{i}Y_n}{2} \left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i \right)$ (X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

"Jordan-Wigner transformation"

[Jordan-Wigner'28]

$$\Downarrow$$

$$H = J \sum_{n=0}^{N-2} \left[q \sum_{i=0}^{n} \frac{Z_i + (-1)^i}{2} + \frac{\vartheta_n}{2\pi} \right]^2 + \frac{w}{2} \sum_{n=0}^{N-2} \left[X_n X_{n+1} + Y_n Y_{n+1} \right] + \frac{m}{2} \sum_{n=0}^{N-1} (-1)^n Z_n$$

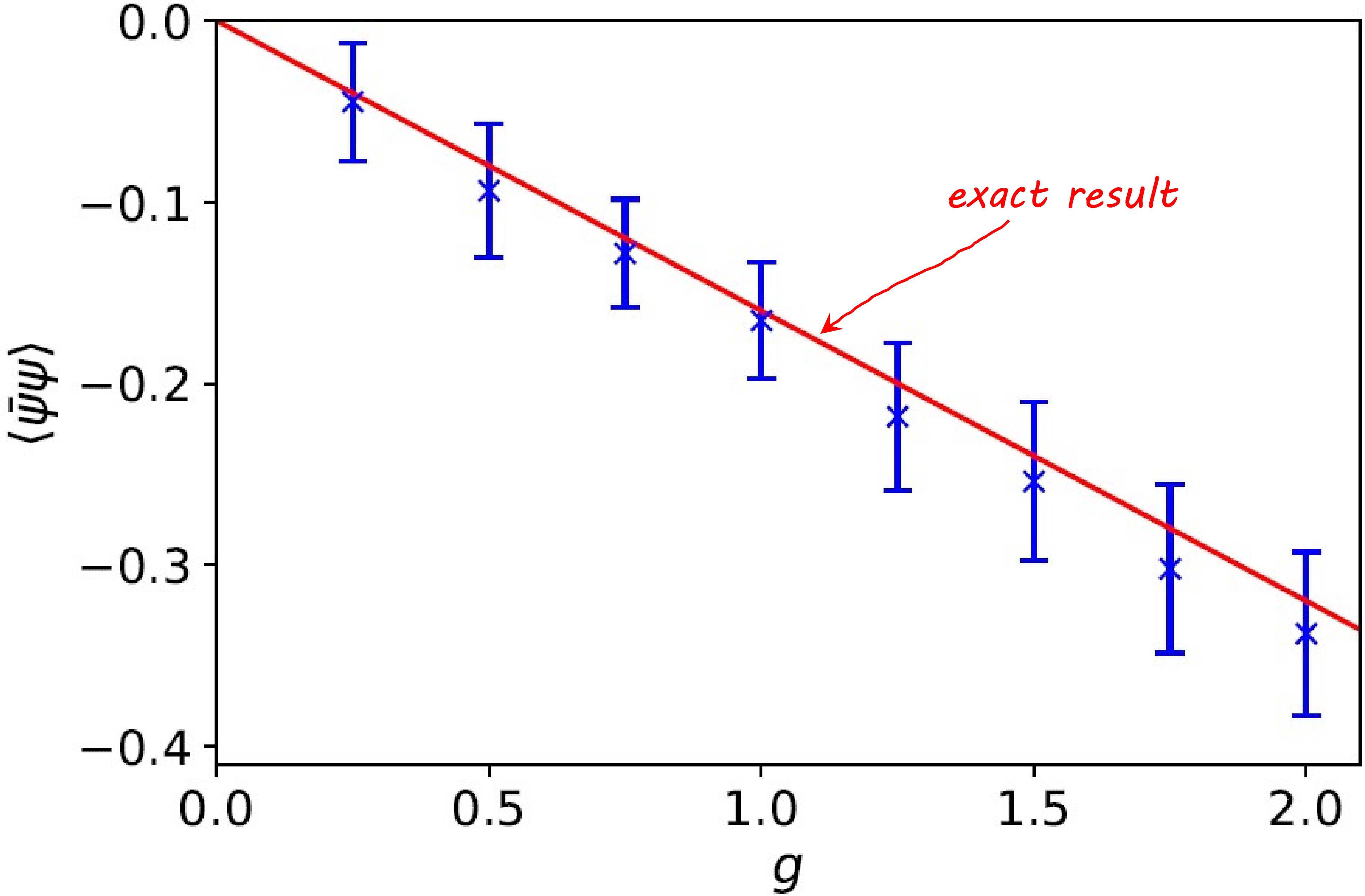
Qubit description of the Schwinger model !!

Ground state expectation value in massless case

$T = 100, \delta t = 0.1, N_{\max} = 16, 1M$ shots

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

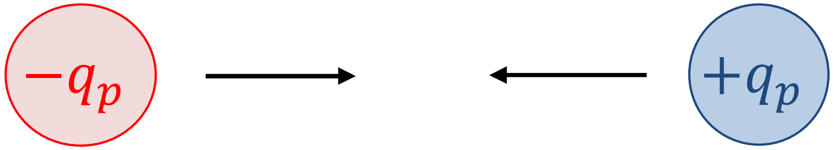
(after continuum limit)



Screening versus Confinement

Let's consider

potential between 2 heavy charged particles



Classical picture:

$$V(x) = \frac{q_p^2\, g^2}{2} x \;?$$

Coulomb law in 1+1d
||
confinement

too naive in the presence of dynamical fermions

Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}(1 - e^{-q\mu x})$$

screening

- massive case:

Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}(1 - e^{-q\mu x})$$ *screening*

- massive case: [cf. Misumi-Tanizaki-Unsal '19]

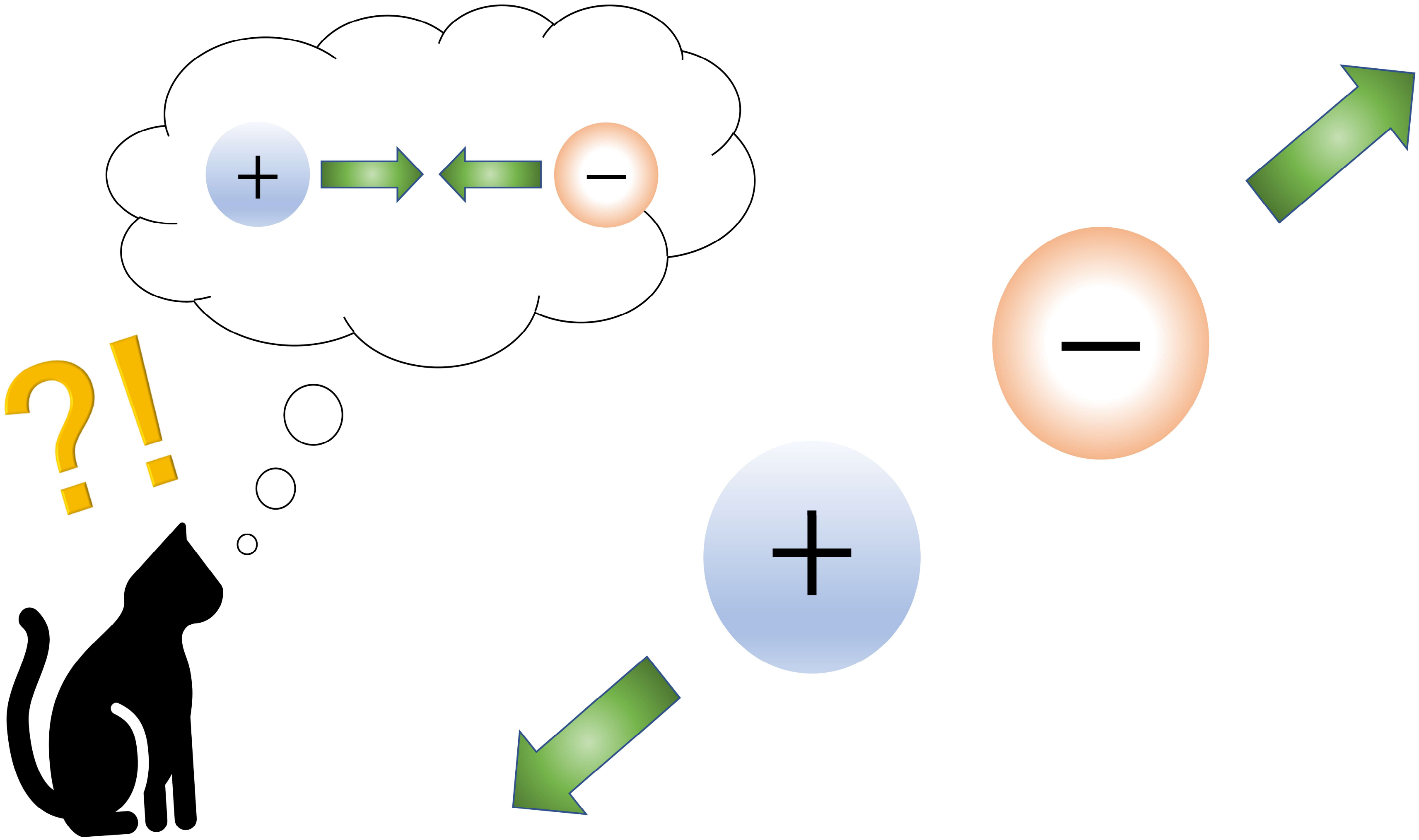
$\Sigma \equiv g e^{\gamma}/2\pi^{3/2}$

$$V(x) \sim mq\Sigma\left(\cos\left(\frac{\theta + 2\pi q_p}{q}\right) - \cos\left(\frac{\theta}{q}\right)\right) x$$ $(\mathrm{m} \ll g,\ |x| \gg 1/g\ )$

$$\begin{cases} = \text{Const.} & \text{for } \mathrm{q_p/q} = \mathbf{Z} \quad \textit{screening} \\ \propto x & \text{for } \mathrm{q_p/q} \neq \mathbf{Z} \quad \textit{confinement?} \end{cases}$$

but sometimes negative slope!

That is, as changing the parameters...



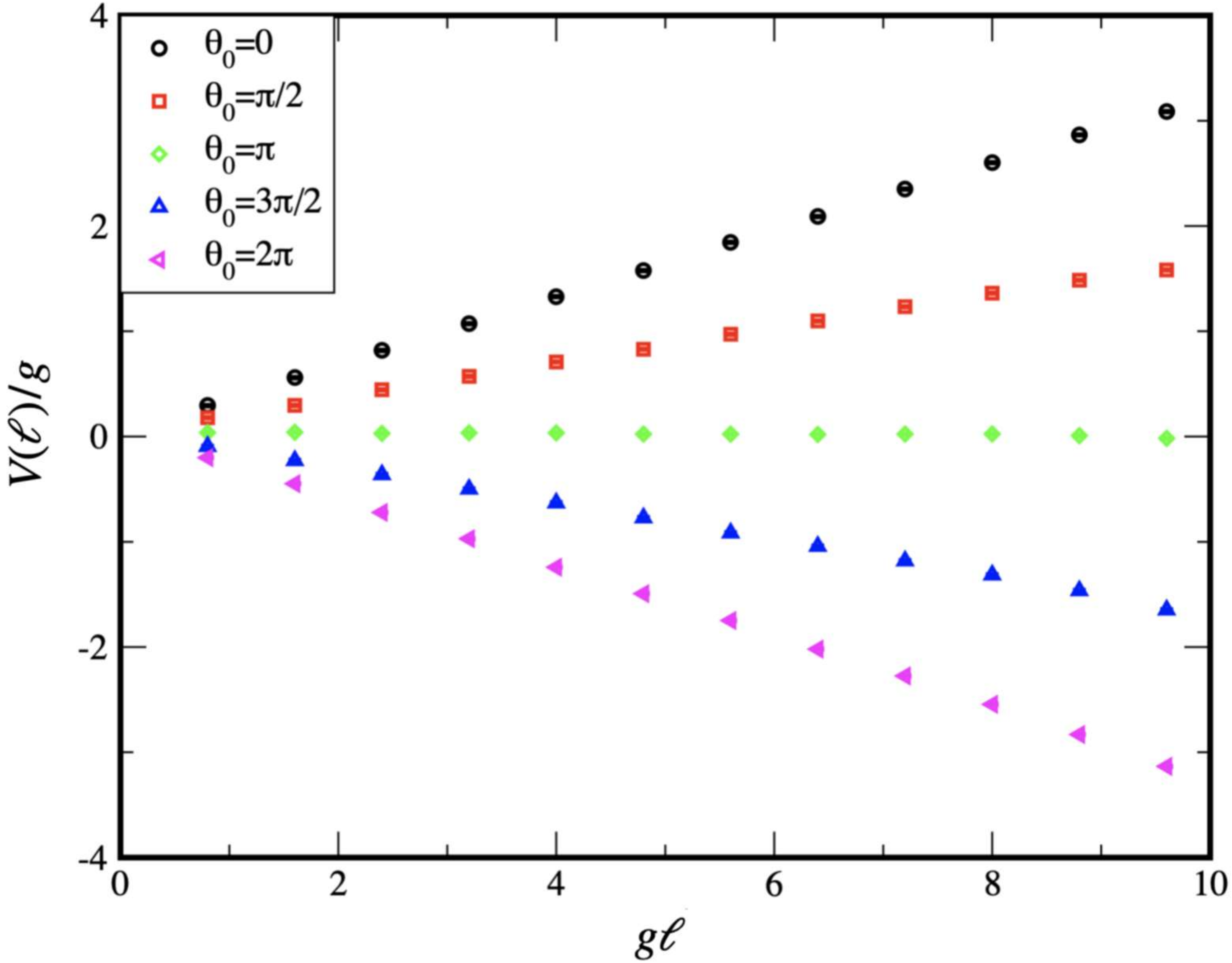
Let’s explore this aspect by quantum simulation!

Positive / negative string tension

[MH-Itou-Kikuchi-Tanizaki '21]

[cf. MH-Itou-Kikuchi-Nagano-Okuda '21]

Parameters: $g = 1, a = 0.4, N = 25, T = 99, q_p/q = -1/3, m = 0.15$

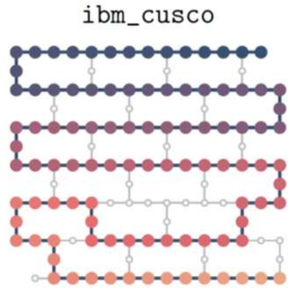


Sign(tension) changes as changing θ-angle!!

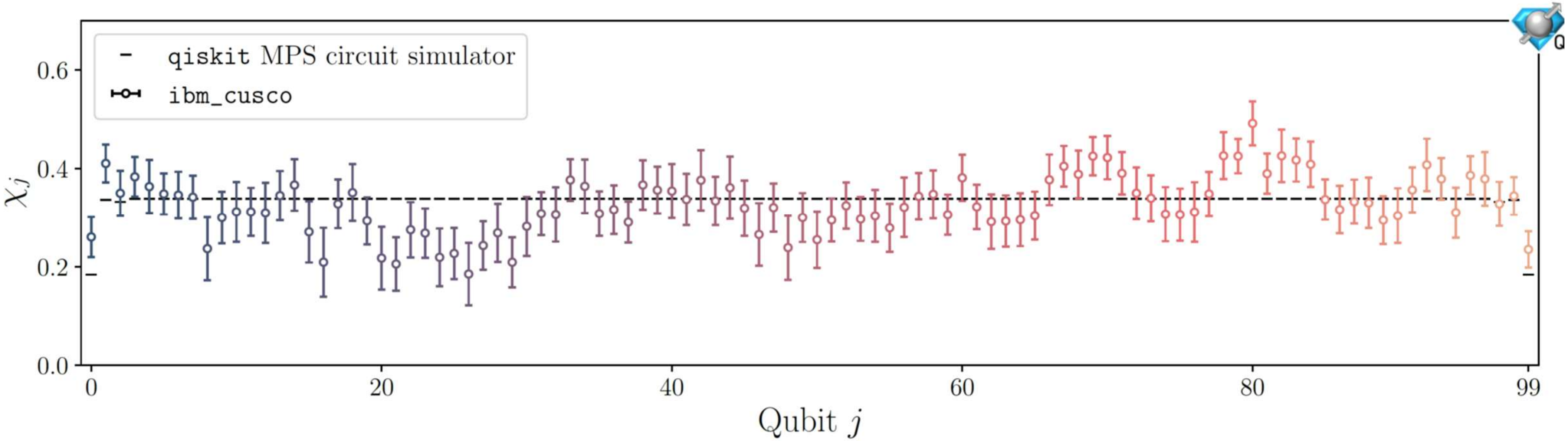
100 qubit simulation of Schwinger model

(127-qubit device: **ibm_cusco** w/ error mitigation)

[Farrel-Illa-Ciavarella-Savage ’23]



Ground state exp. of local chiral condensate :



Other simulations of Schwinger model

- decay of massive vacuum under time evolution

[cf. Martinez etal. **Nature** 534 (2016) 516-519]

- quenched dynamics of θ [Nagano-Bapat-Bauer ’23]

- Schwinger model in open quantum system

[De Jong-Metcalf-Mulligan-Ploskon-Ringer-Yao ’20, de Jong-Lee-Mulligan-Ploskon-Ringer-Yao ’21, Lee-Mulligan-Ringer-Yao ’23]

- 112 qubit simulation of meson propagation

[Farrell-Illa-Ciavarella-Savage ’24]

- finding energy spectrum [MH-Ghim, work in progress]

- finite temperature [Itou-Sun-Pedersen-Yunoki ’23]

etc...

“Scattering” in Thirring model

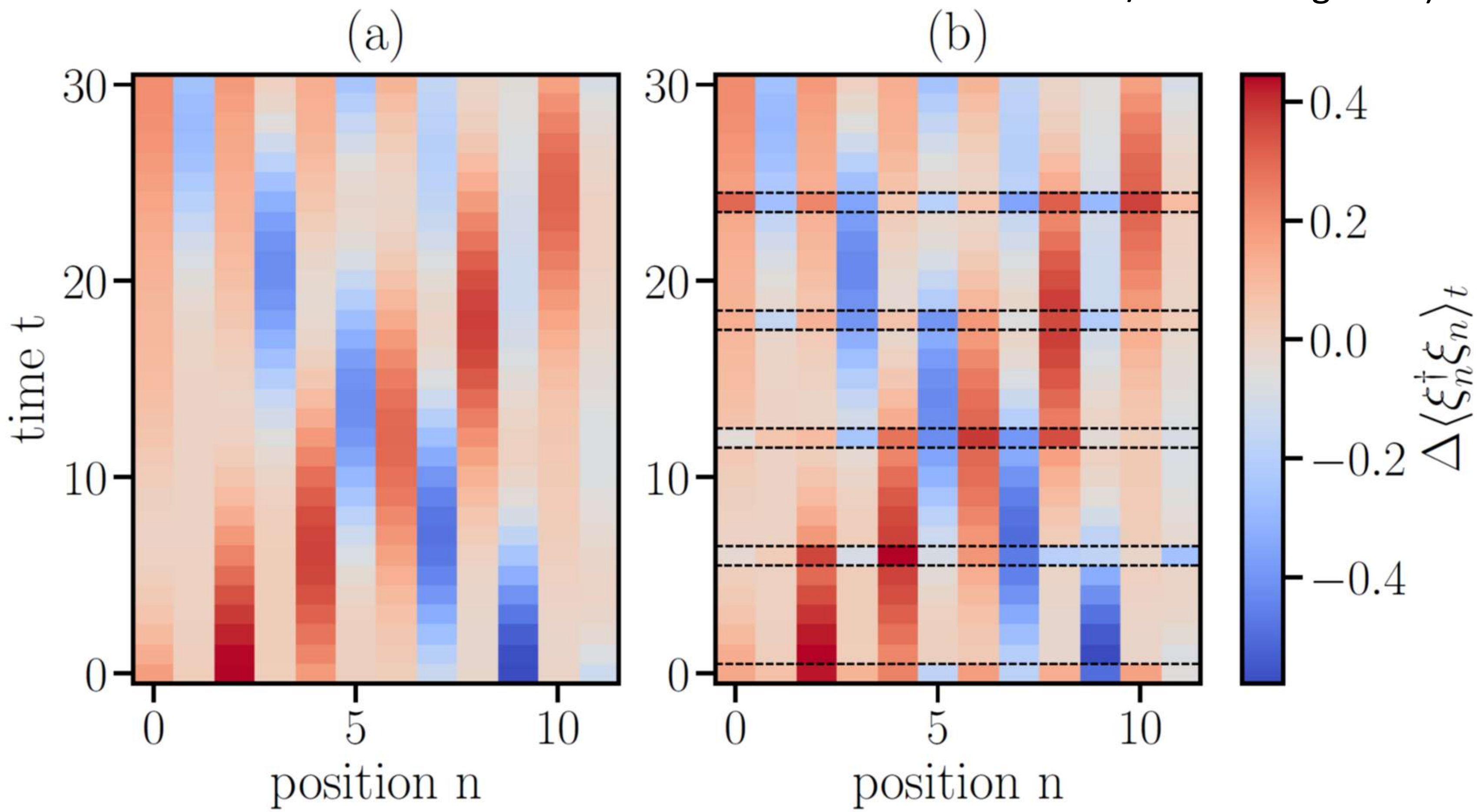
Thirring model on lattice:

[Chai-Crippa-Jansen-Kuhn-Pascuzzi-Tacchino-Tavernelli ’23]

$$H = \sum_{n=0}^{N-1} \left(\frac{i}{2a} \left(\xi_{n+1}^\dagger \xi_n - \xi_n^\dagger \xi_{n+1} \right) + (-1)^n m \, \xi_n^\dagger \xi_n \right) + \sum_{n=0}^{N-1} \frac{g(\lambda)}{a} \xi_n^\dagger \xi_n \xi_{n+1}^\dagger \xi_{n+1},$$

Particle density of two wave packets:

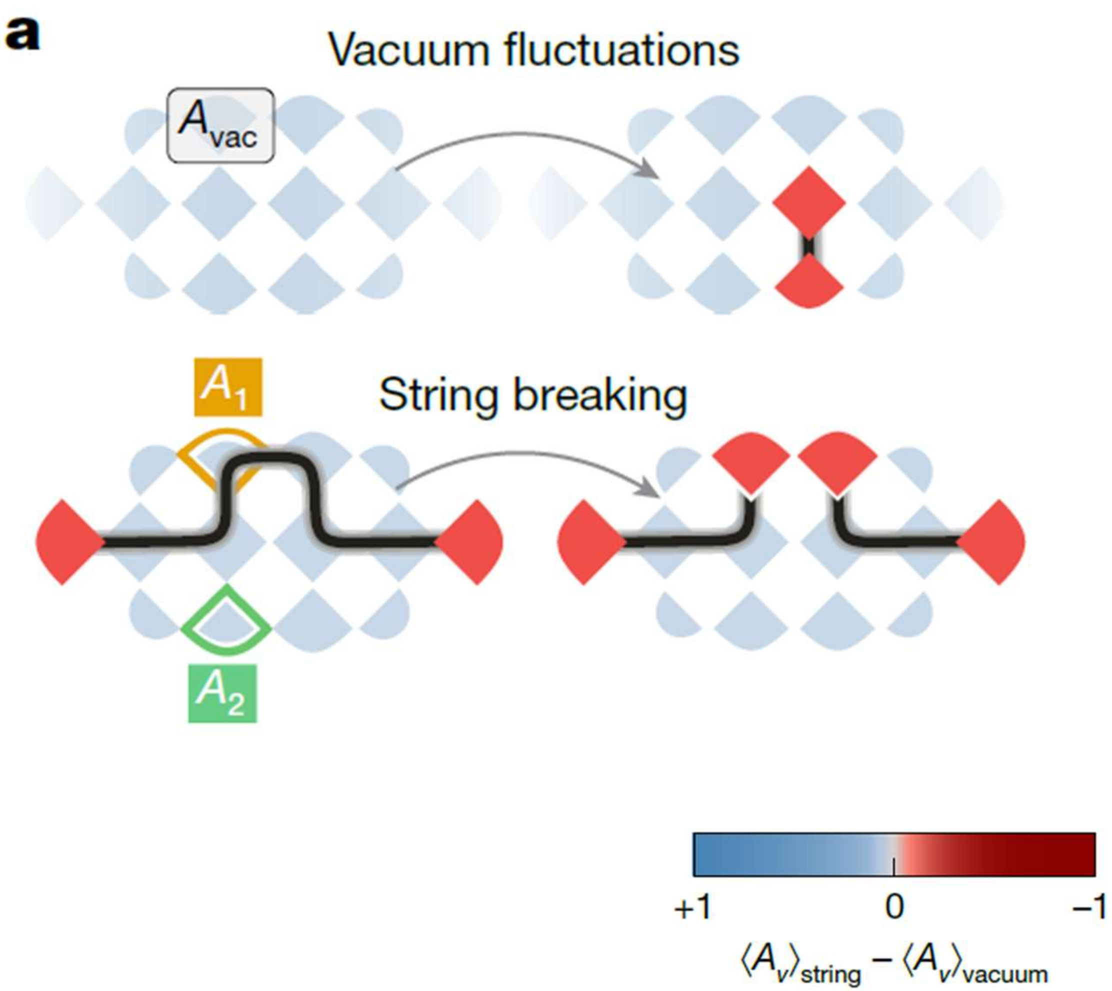
(12-qubit device: **ibm_peekskill** w/ error mitigation)

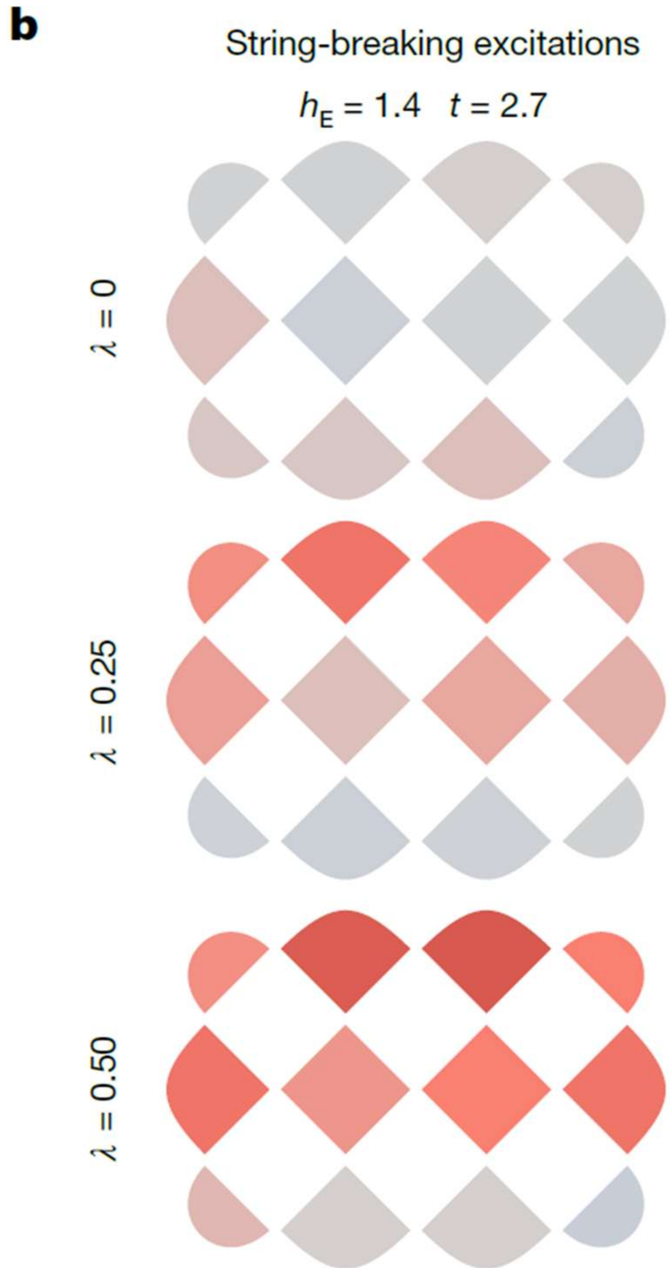


String breaking in 2+1d Z_2 gauge theory (?)

[Simulation by 72 qubit Google Sycamore '24]

$$\mathcal{H} = -J_{\rm E}\sum_v A_v - J_{\rm M}\sum_p B_p - h_{\rm E}\sum_{\rm links} Z_l - \lambda\sum_{\rm links} X_l.$$





On higher dimensional fermion

Go to higher dimensions! [MH, work in progress]

$\Downarrow$

1st step: find a nice way to map 2d fermion to spins

Problem in naïve approach:

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i\right)$$

- 1d

$$\chi_{n+1}^{\dagger}\chi_n \xrightarrow{\text{Jordan-Wigner}} {}^{\exists}X_{n+1}X_n, Y_{n+1}Y_n, X_{n+1}Y_n, Y_{n+1}X_n$$

local

- 2d

On higher dimensional fermion

Go to higher dimensions! [MH, work in progress]

$\Downarrow$

1st step: find a nice way to map 2d fermion to spins

Problem in naïve approach:

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i\right)$$

▪ 1d

$$\chi_{n+1}^\dagger \chi_n \xrightarrow{\text{Jordan-Wigner}} {}^{\exists}X_{n+1}X_n, Y_{n+1}Y_n, X_{n+1}Y_n, Y_{n+1}X_n$$

local

▪ 2d ($N \times N$ square lattice)

Relabeling site (i, j) like 1d label (say $n = i + Nj$),

$$\chi_{(i,j+1)}^\dagger \chi_{(i,j)} = \chi_{I+N}^\dagger \chi_I \xrightarrow{\text{JW}} {}^{\exists}X_{I+N}X_I \prod_{i=I+1}^{I+N-1} Z_i \text{ , etc...}$$

(cf. $\mathcal{O}(\log N)$ for Bravyi-Kitaev trans.) *non-local*

On non-abelian gauge theory

$^\exists$Various approaches but not sure which is better

- truncation of electric field [Byrnes-Yamamoto '05, etc...]
- truncation of representations
- discrete group [Gustafson-Ji-Lamm-Murairi-Perez'24, etc...]
- quantum group [Zache-Gonzalez-Cuadra-Zoller '23, Hayata-Hidaka '23]
- orbifold lattice [Buser-Gharibyan-Hanada-MH-Liu '20, etc...]
- fuzzy gauge theory [Alexandru-Bedaque-Carosso-Cervia-Murairi-Sheng '24]

Contents

Quantum Gravity on Quantum Computer

(QG) (QC)

<u>Most difficult point:</u>

We don't know what (realistic) QG is…

<u>Approaches:</u>

Quantum Gravity on Quantum Computer

(QG) (QC)

<u>Most difficult point:</u>

We don't know what (realistic) QG is…

<u>Approaches:</u>

1. study situations w/ known formulations
   — e.g. (1+1) & (2+1) dimensions
2. assume hypothetical formulations & use them
   — e.g. loop QG, dynamical triangulation, etc…
3. study systems (hypothetically) equivalent to QG
   — e.g. holography, matrix model

Example of holography approach

nature

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nature > articles > article

Article | Published: 30 November 2022

Traversable wormhole dynamics on a quantum processor

Daniel Jafferis, Alexander Zlokapa, Joseph D. Lykken, David K. Kolchmeyer, Samantha I. Davis, Nikolai Lauk, Hartmut Neven & Maria Spiropulu

[Submitted on 15 Feb 2023]

Comment on "Traversable wormhole dynamics on a quantum processor"

Bryce Kobrin, Thomas Schuster, Norman Y. Yao

[Submitted on 27 Mar 2023]

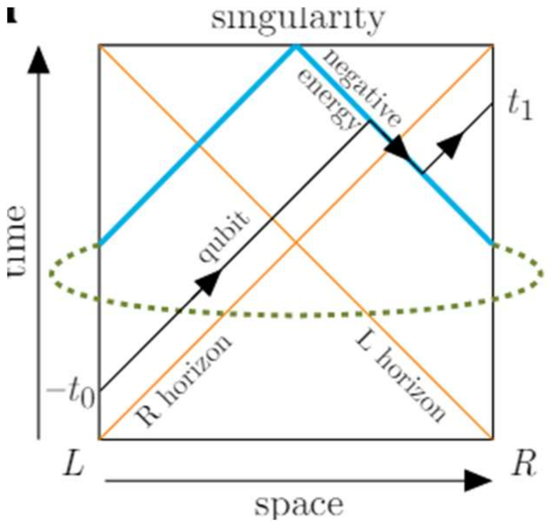
Comment on "Comment on "Traversable wormhole dynamics on a quantum processor" "

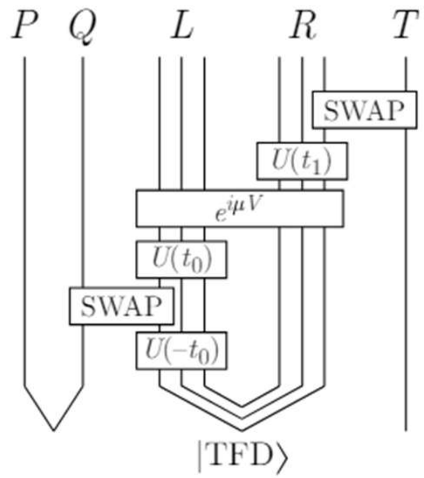
Daniel Jafferis, Alexander Zlokapa, Joseph D. Lykken, David K. Kolchmeyer, Samantha I. Davis, Nikolai Lauk, Hartmut Neven, Maria Spiropulu

Example of holography approach

“Wormhole experiment” by Google Sycamore:

[Nature, Jafferis-Zlokapa-Lykken-Kolchmeyer-Davis-Lauk-Neven-Spiropulu ’23]





- simulation of sparse SYK model assuming holography for JT gravity
- based on various nontrivial assumptions

  → contravarcy on whether wormhole was really made

Can we make it directly on the gravity side?

Example of direct approach

Jackiw-Teitelboim gravity (JT gravity) :

[work in progress, MH]

$$I_{\mathrm{JT}} = \int_M d^2x\,\sqrt{-g}\,\Phi(R+2) + 2\int_{\partial M}\sqrt{|\gamma|}\,\Phi(K-1) + \cdots$$

(R:curvature, Φ: scalar)

Example of direct approach

Jackiw-Teitelboim gravity (JT gravity) :

[work in progress, MH]

$$I_{\mathrm{JT}} = \int_M d^2x\,\sqrt{-g}\,\Phi(R+2) + 2\int_{\partial M}\sqrt{|\gamma|}\,\Phi(K-1) + \cdots$$

(R:curvature, Φ: scalar)

Switching to operator formalism & solving physical conditions, the analysis is boiled down to a one-particle quantum mechanics:

[Jafferis-Kolchmeyer '19]

$$\boxed{H_{\mathrm{JT}} = \frac{p^2}{2} + \frac{1}{2}e^{-x}}$$

We can simulate the JT gravity by simulating this!

Note: this is exactly solvable QM but we can also formulate JT gravity coupled to matter in a similar way which is not exactly solvable

How to simulate wormhole physics

[work in progress, MH]

1. Truncation by cutoff Λ:

$$H_{\mathrm{JT}} = \frac{p^2}{2} + \frac{1}{2}e^{-x} \xrightarrow{\text{truncation}} H_\Lambda$$

2. Construct a Hartle-Hawking state

$$\left|\Psi_\beta^\Lambda\right\rangle := \sum_{n=0}^{\Lambda} f_\beta(E_n)|E_n\rangle$$

※can be constructed in a similar way to imaginary time evolution

[cf. Kosugi-Nishiya-Nishi-Matsushita '21]

3. Look at time evolution of survival probability

$$P(t) := \left|\left\langle\Psi_\beta^\Lambda\middle|e^{-iH_\Lambda t}\middle|\Psi_\beta^\Lambda\right\rangle\right|^2$$

(wormhole contributions appear as exponential decay as a function of coupling)

Example of matrix model approach

Matrix Quantum Mechanics (QM)

|| literally

QM of matrices

Ex.) One Hermitian matrix QM: $(X(t)$: Hermitian matrix)

- Path integral formalism

$$L = \mathrm{Tr}\left[\frac{1}{2}\dot{X}^2 - V(X)\right], \qquad Z = \int DX\, e^{i\int dt\, L}$$

- Operator formalism

$$H = \mathrm{Tr}\left[\frac{1}{2}P^2 + V(X)\right], \qquad \left[X_{ij}\, , P_{k\ell}\right] = i\delta_{ik}\delta_{j\ell}$$

Technically,

special case of many particle QM

BMN matrix model ($U(N)$ gauged matrix QM)

[Berenstein-Maldacena-Nastase '02]

$$L = \frac{1}{g^2}\mathrm{Tr}\left\{\frac{1}{2}(D_t X_I)^2 + \frac{1}{4}[X_I, X_J]^2 - \frac{\mu^2}{18}X_i^2 - \frac{\mu^2}{72}X_a^2 - \frac{i\mu}{6}\epsilon^{ijk}X_i X_j X_k \right.$$
$$\left. + \frac{i}{2}\Psi^\dagger D_t \Psi - \frac{1}{2}\Psi^\dagger \gamma_I [X_I, \Psi] - \frac{i\mu}{8}\Psi^\dagger \gamma_{123}\Psi \right\},$$

- (0+1) dim. $U(N)$ gauge theory
- all the fields are $N \times N$ Hermitian matrices
- X_I: bosonic matrices ($I = 1, \cdots, 9$)
- Ψ: 16 component Majorana-Weyl fermion
- $i = 1,2,3, a = 4, \cdots, 9$

BMN matrix model (cont'd)

[Berenstein-Maldacena-Nastase '02]

$$L = \frac{1}{g^2}\mathrm{Tr}\left\{\frac{1}{2}(D_t X_I)^2 + \frac{1}{4}[X_I, X_J]^2 - \frac{\mu^2}{18}X_i^2 - \frac{\mu^2}{72}X_a^2 - \frac{i\mu}{6}\epsilon^{ijk}X_i X_j X_k + \frac{i}{2}\Psi^\dagger D_t \Psi - \frac{1}{2}\Psi^\dagger \gamma_I [X_I, \Psi] - \frac{i\mu}{8}\Psi^\dagger \gamma_{123}\Psi\right\},$$

related to various interesting "stringy" theories:

- M-theory on pp-wave spacetime
- 3d $\mathcal{N} = 8$ SYM on $\boldsymbol{R} \times S^2$ ~ D2-branes in IIA string theory
- 4d $\mathcal{N} = 4$ SYM on $\boldsymbol{R} \times S^3$ ~ D3-branes in IIB string theory

[Ishii-Ishiki-Shimasaki-Tsuchiya '08, etc…]

- 6d $\mathcal{N} = (2,0)$ theory on $\boldsymbol{R} \times S^5$ ~ M5-branes in M-theory

[Maldacena-Sheikh-Jabbari-Van Raamsdonk '02]

- holographic duals

Operator formalism

$$\Psi = \begin{pmatrix} \psi_{Ip} \\ \epsilon_{pq}\psi^{\dagger Iq} \end{pmatrix}$$

$$\hat{H} = \mathrm{Tr}\Big\{ \frac{1}{2}(\hat{P}_I)^2 - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2 + \frac{\mu^2}{18}\hat{X}_i^2 + \frac{\mu^2}{72}\hat{X}_a^2 + \frac{i\mu g}{3}\epsilon^{ijk}\hat{X}_i\hat{X}_j\hat{X}_k$$

$$+ g\hat{\psi}^{\dagger Ip}\sigma_p^{iq}[\hat{X}_i, \hat{\psi}_{Iq}] - \frac{g}{2}\epsilon_{pq}\hat{\psi}^{\dagger Ip}g^a_{IJ}[\hat{X}_a, \hat{\psi}^{\dagger Jq}] + \frac{g}{2}\epsilon^{pq}\hat{\psi}_{Ip}(g^{a\dagger})^{IJ}[\hat{X}_a, \hat{\psi}_{Jq}] + \frac{\mu}{4}\hat{\psi}^{\dagger Ip}\hat{\psi}_{Ip} \Big\}.$$

Commutation relations:

(α, β: gauge indices)

$$\left[\hat{X}_{I\alpha}, \hat{P}_{J\beta}\right] = i\delta_{IJ}\delta_{\alpha\beta}\,, \qquad \left\{\hat{\psi}^{\dagger Ip\alpha}, \hat{\psi}^{\beta}_{Jq}\right\} = \delta_{IJ}\delta^{pq}\delta^{\alpha\beta}$$

Gauss law:

$$\hat{G}_\alpha|\mathrm{phys}\rangle = 0 \quad \text{w/} \quad \hat{G}_\alpha = \sum_{\beta,\gamma=1}^{N^2}\left(\sum_{I=1}^{9}\hat{X}_I^\beta\hat{P}_I^\gamma - i\sum_{I,p}\hat{\psi}^{\dagger Ip\alpha}\hat{\psi}^\gamma_{Ip}\right)$$

We can regularize it as in scalar field theory

[Gharibyan-Hanada-MH-Liu '20]

Computational costs

of qubits:

- Single particle QM w/ truncation Λ requires $\log_2 \Lambda$ qubits
- The BMN model has 9 scalars & 16 component real fermion which are $N \times N$ matrices

$$\Rightarrow \quad 9N^2 \log_2 \Lambda + 8N^2 \ \text{ qubits}$$

of spin ops. in Hamiltonian:

- each annihilation/creation op. has less than $\mathcal{O}(\Lambda^2)$ spin ops.
- we have 4-pt. interaction at most
- $^\exists\mathcal{O}(N^4)$ combinations regarding the color indices

$$\Rightarrow \quad < \mathcal{O}(\Lambda^8 N^4) \text{ spin ops.}$$

of qubits to simulate black hole

BMN w/ truncation has [Maldacena '23]

$$\boxed{9N^2 \log_2 \Lambda + 8N^2 \ \text{qubits}}$$

What N & Λ needed to simulate black hole?

- MC study suggests BH entropy is (approximately) reproduced at

$$N = 16, \frac{T}{(g^2 N)^{1/3}} = 0.3, \frac{\mu}{T} = 1.6$$

[Patelpudis-Bergner-Hanada-Rinaldi-Schafer -Vranas-Watanabe-Bpdendorfer '22]

- Important energy levels should satisfy about $E_n < \mathcal{O}(T)$

$$\longrightarrow \quad \Lambda \sim 4$$

Totally, we need

$$\boxed{\sim 7000 \ \text{qubits}}$$

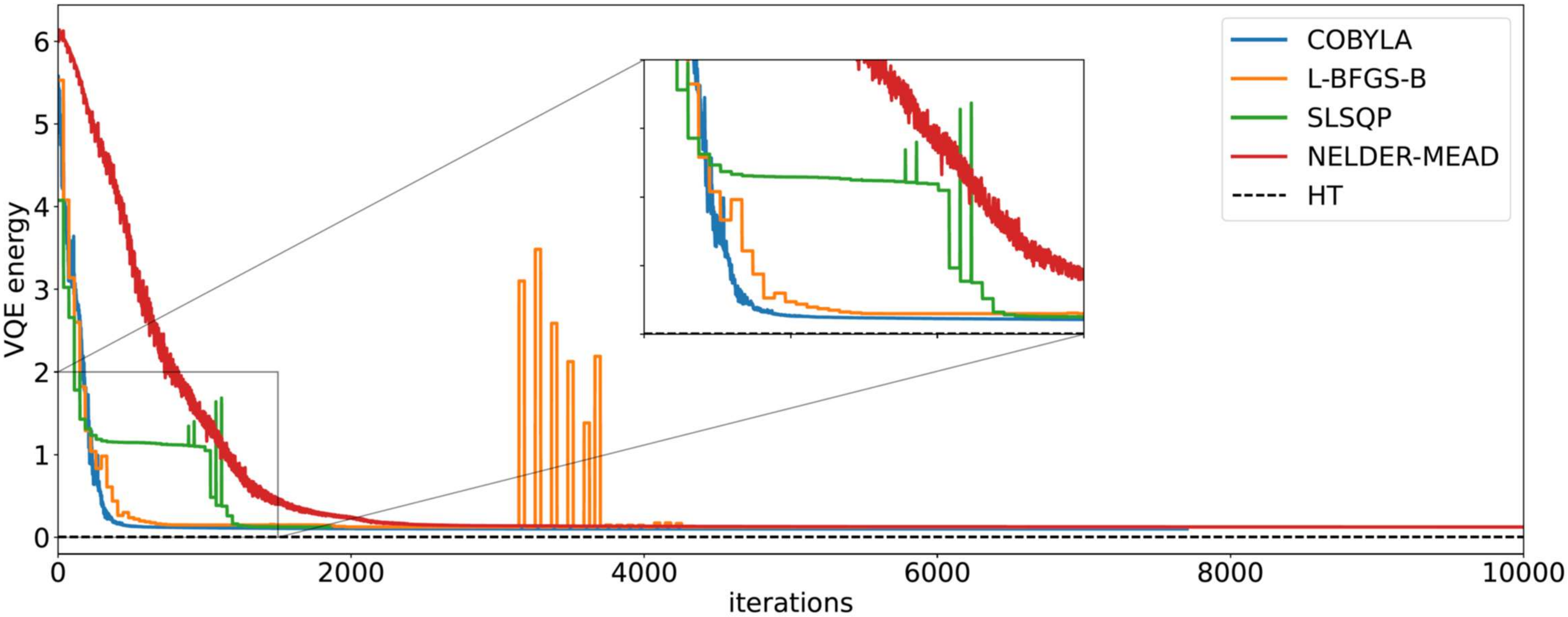
(similar to the condition for "quantum supremacy" in factoring integer)

An implementation for “$SU(2)$ mini-BMN”

[Rinaldi-Han-Hassan-Feng-Nori-McGuigan-Hanada ’21]

$$\hat{H} = \mathrm{Tr}\left(\frac{1}{2}\hat{P}_I^2 - \frac{g^2}{4}[\hat{X}_I, \hat{X}_J]^2 + \frac{g}{2}\hat{\bar{\psi}}\Gamma^I[\hat{X}_I, \hat{\psi}] - \frac{3i\mu}{4}\hat{\bar{\psi}}\hat{\psi} + \frac{\mu^2}{2}\hat{X}_I^2\right) - (N^2-1)\mu$$

Ground state energy by VQE on simulator $(\Lambda = 2)$



Outlook

Near future prospect

In near future, available device is so-called [Preskill '18]

Noisy intermediate-scale quantum device (NISQ)

w/ limited number of qubits & non-negligible errors

On such device,

・quantum error correction can't be enough

⇨ nice if $^\exists$a way to reduce errors w/o increasing qubits

⇨ "quantum error mitigation"

・algorithms w/ less gates are preferred

⇨ Hybrid quantum-classical algorithm

(Popular one for finding vacuum: "variational method")

Quantum Error mitigation

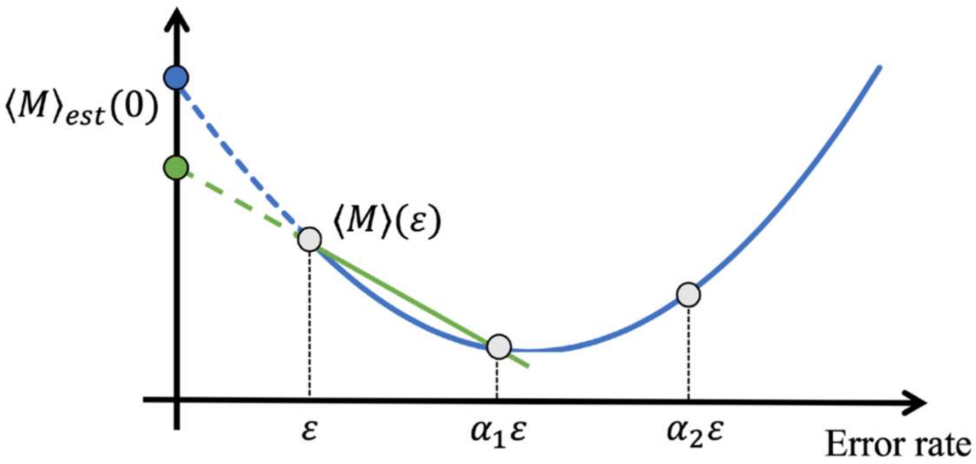
[Figs. are from Endo-Cai-Benjamin-Yuan ’20]

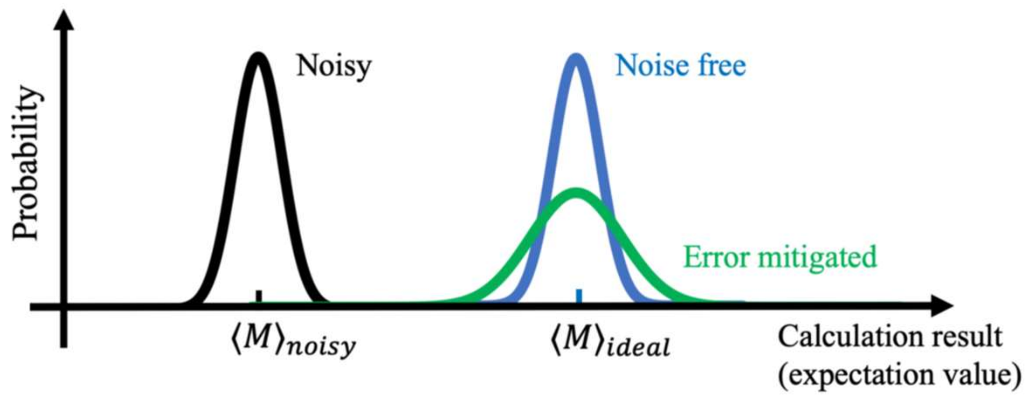
the simplest way = extrapolation

In general,

difficult to decrease errors but possible to increase them

⇨ error-free result by fitting as a function of error rate





This doesn’t need to increase qubits but needs more shots

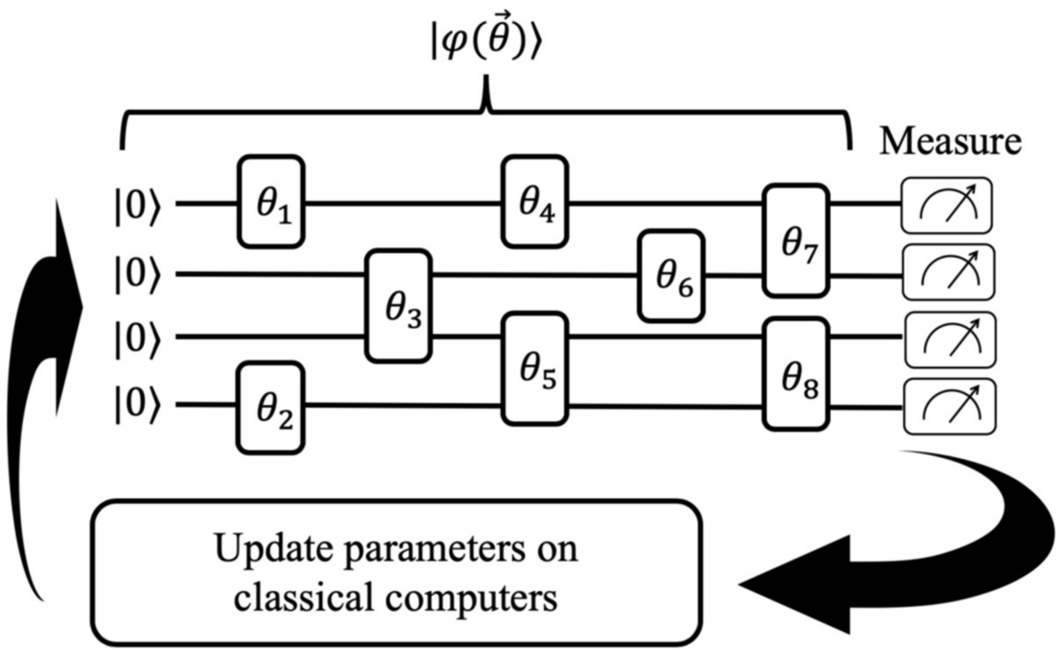
Variational quantum algorithm

[Fig. is from Endo-Cai-Benjamin-Yuan ’20]

Idea:

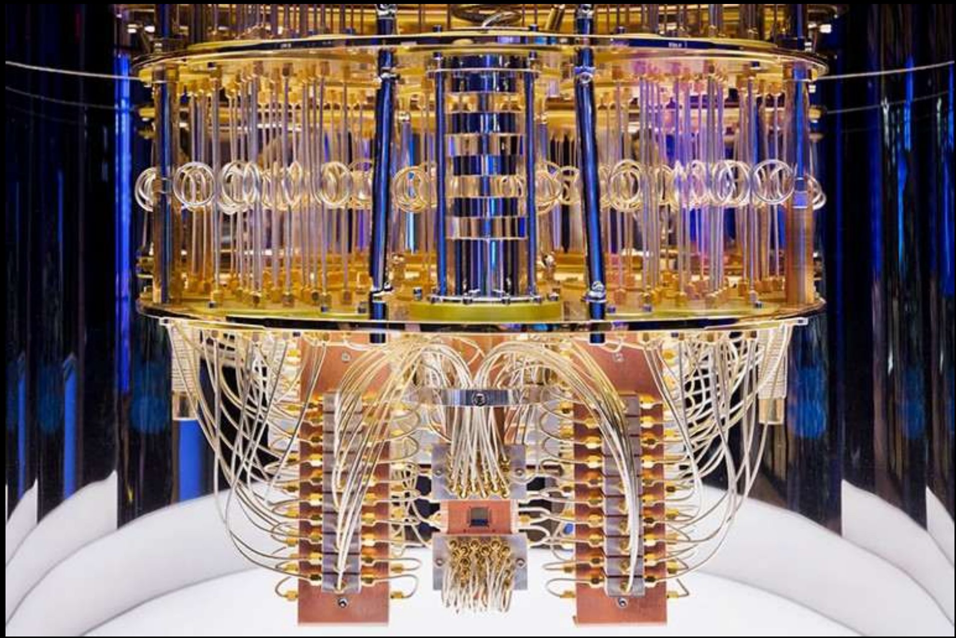
Acting gates & measurements ⇨ Quantum computer

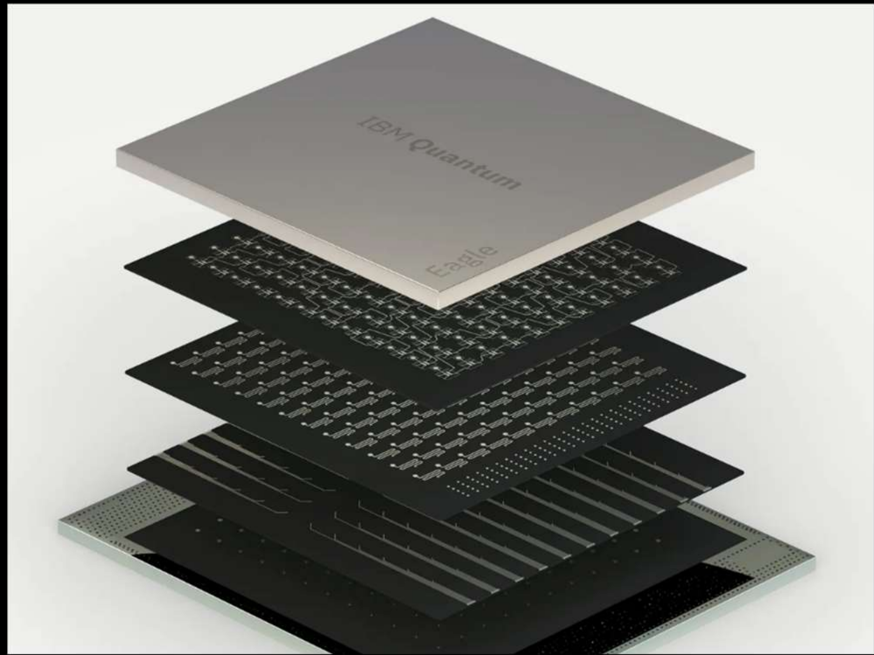
Parameter optimization ⇨ Classical computer



This method needs much less gates than adiabatic state preparation but it’s not guaranteed to get true ground state

The challenge by IBM's 127-qubit device





Article

Evidence for the utility of quantum computing before fault tolerance

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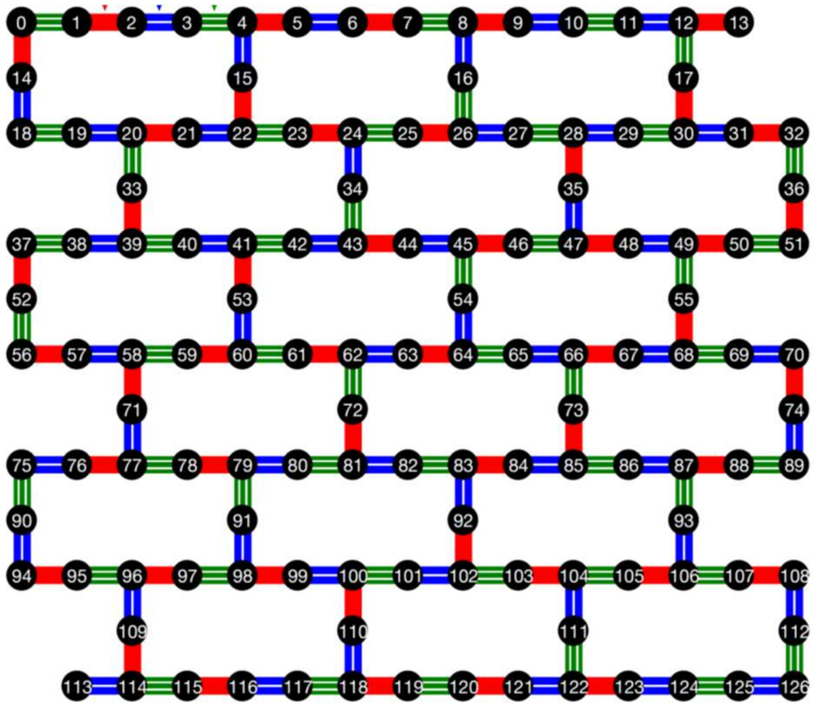
Youngseok Kim[1,6], Andrew Eddins[2,6], Sajant Anand[3], Ken Xuan Wei[1], Ewout van den Berg[1], Sami Rosenblatt[1], Hasan Nayfeh[1], Yantao Wu[3,4], Michael Zaletel[3,5], Kristan Temme[1] & Abhinav Kandala[1]

Quantum computing promises to offer substantial speed-ups over its classical

The challenge by IBM's 127-qubit device (cont'd)

Task: time evolution of Ising model on a lattice
w/ shape = the qubit config. of the device



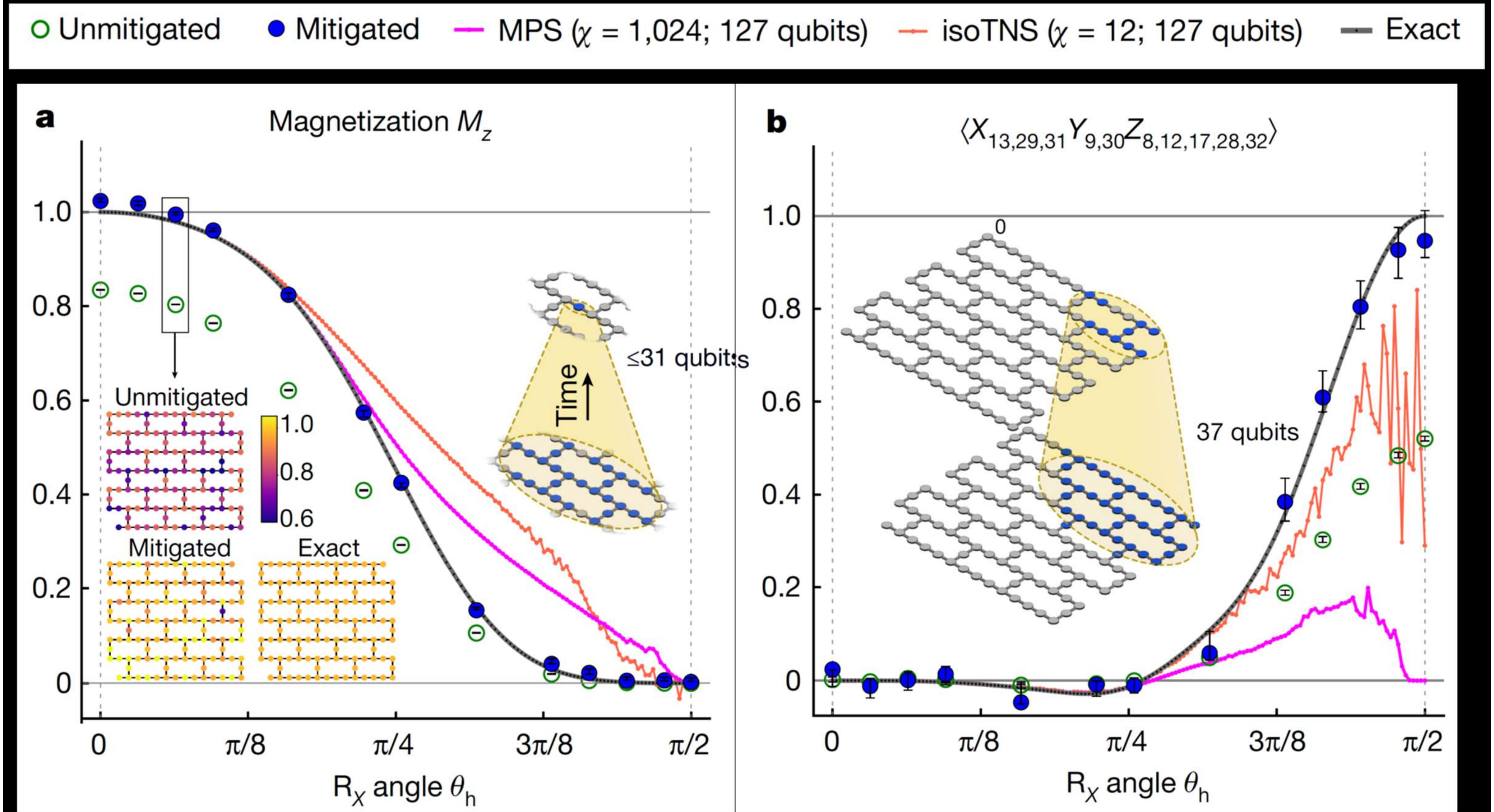
$$H = -J \sum_{\langle i,j \rangle} Z_i Z_j + h \sum_i X_i,$$

$$|\psi(t)\rangle := e^{-iHt} |00 \cdots 0\rangle$$

$$\langle \psi(t) | \mathcal{O} | \psi(t) \rangle$$

Strategy: Suzuki-Trotter approximation
+ error mitigation by extrapolation

The challenge by IBM's 127-qubit device (cont'd)



"Quantum supremacy"?

But…

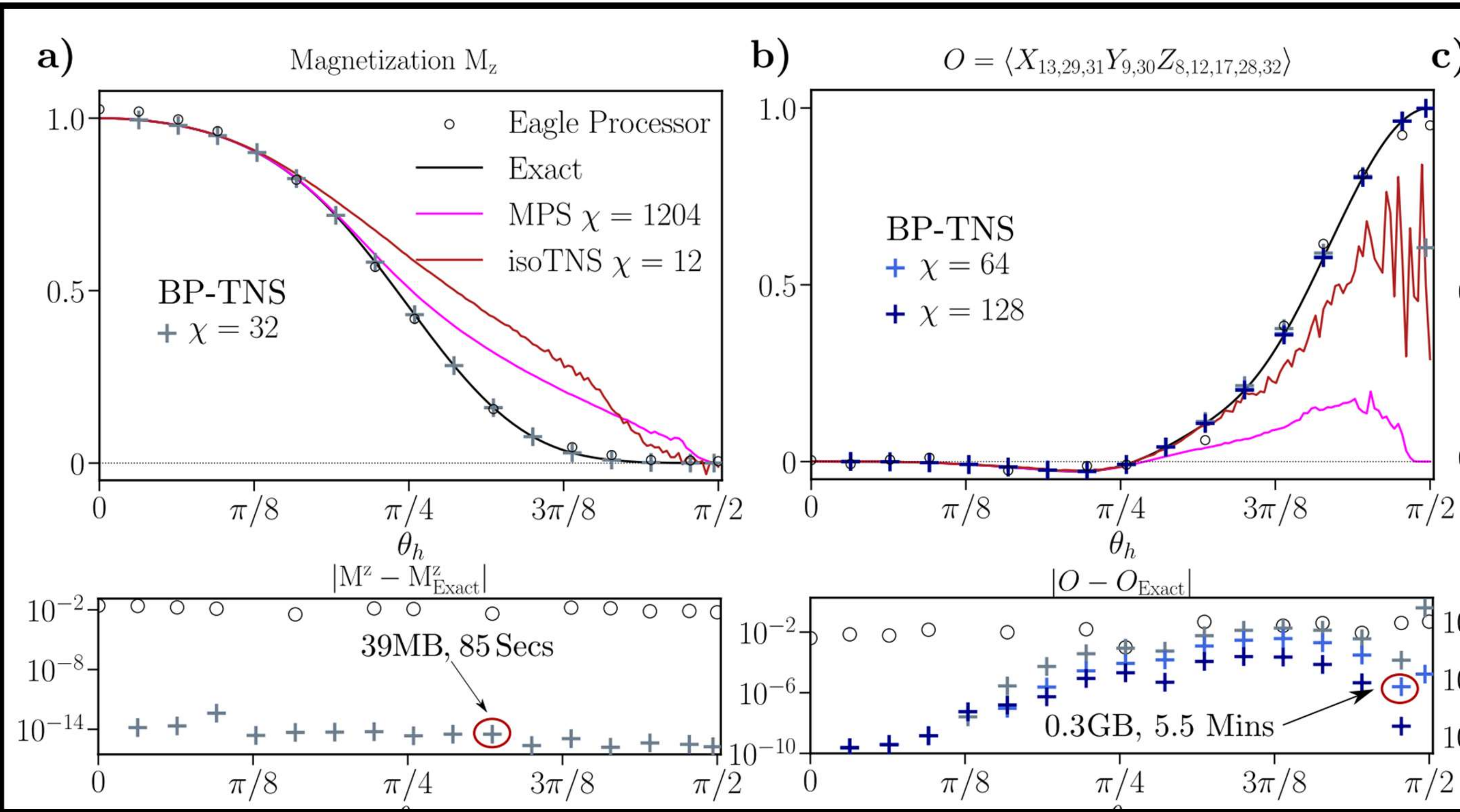
arXiv > quant-ph > arXiv:2306.14887

Quantum Physics

[Submitted on 26 Jun 2023]

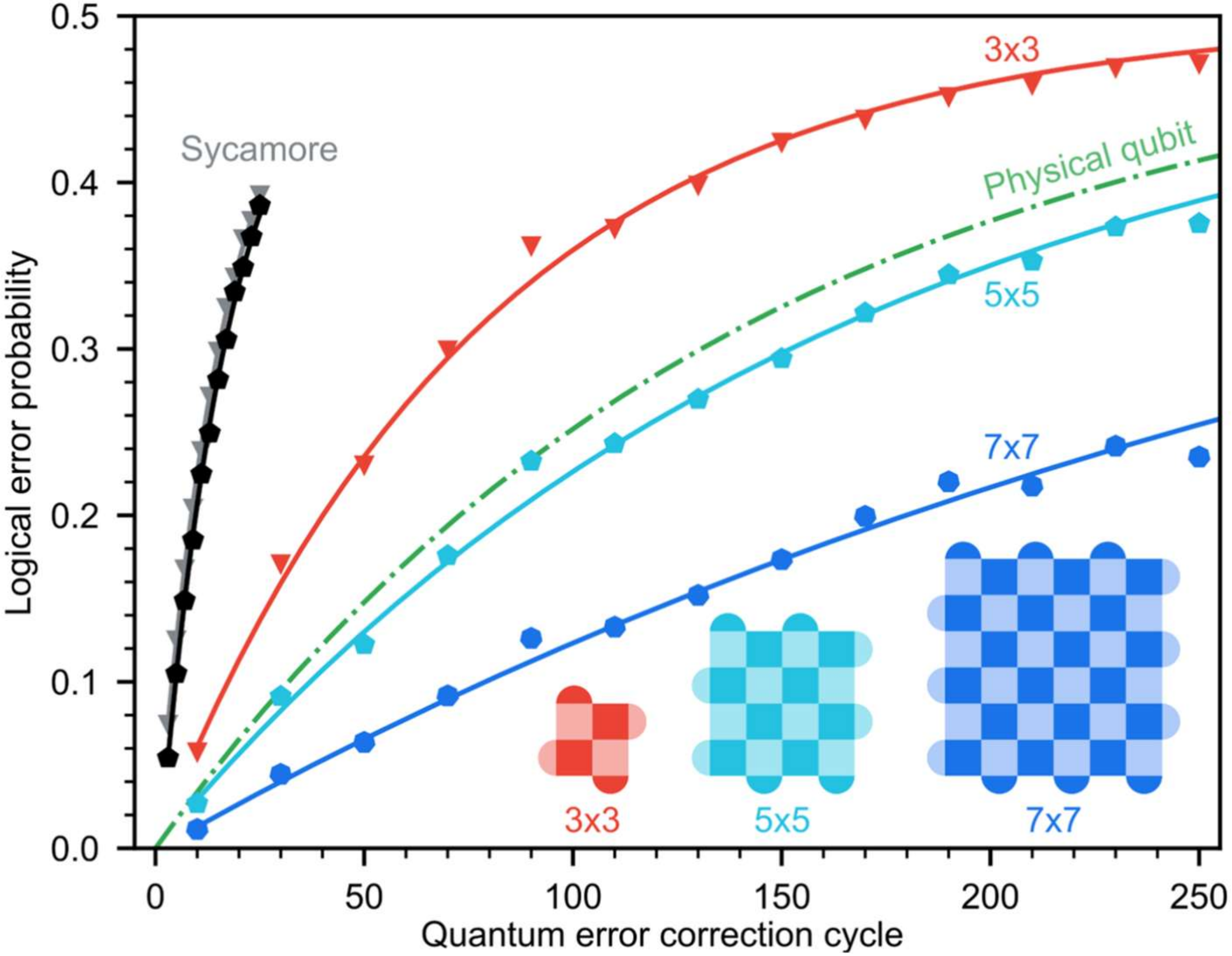
Efficient tensor network simulation of IBM's kicked Ising experiment

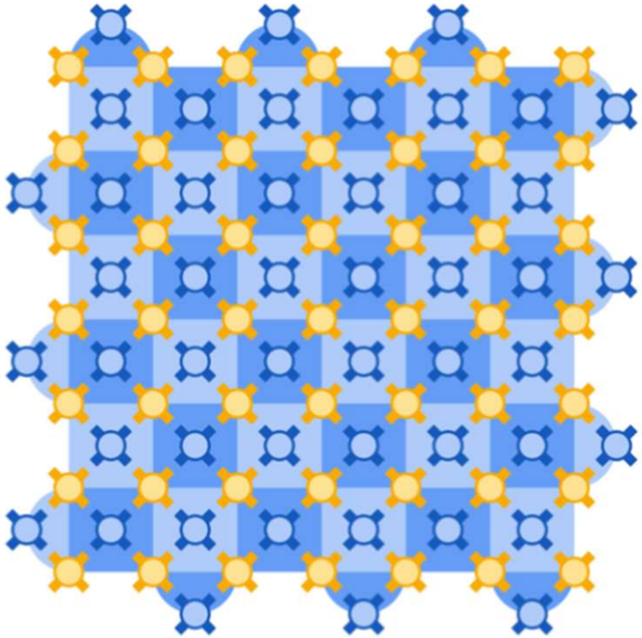
Joseph Tindall, Matt Fishman, Miles Stoudenmire, Dries Sels



Implementation of error correction

[Google Quantum AI '24]



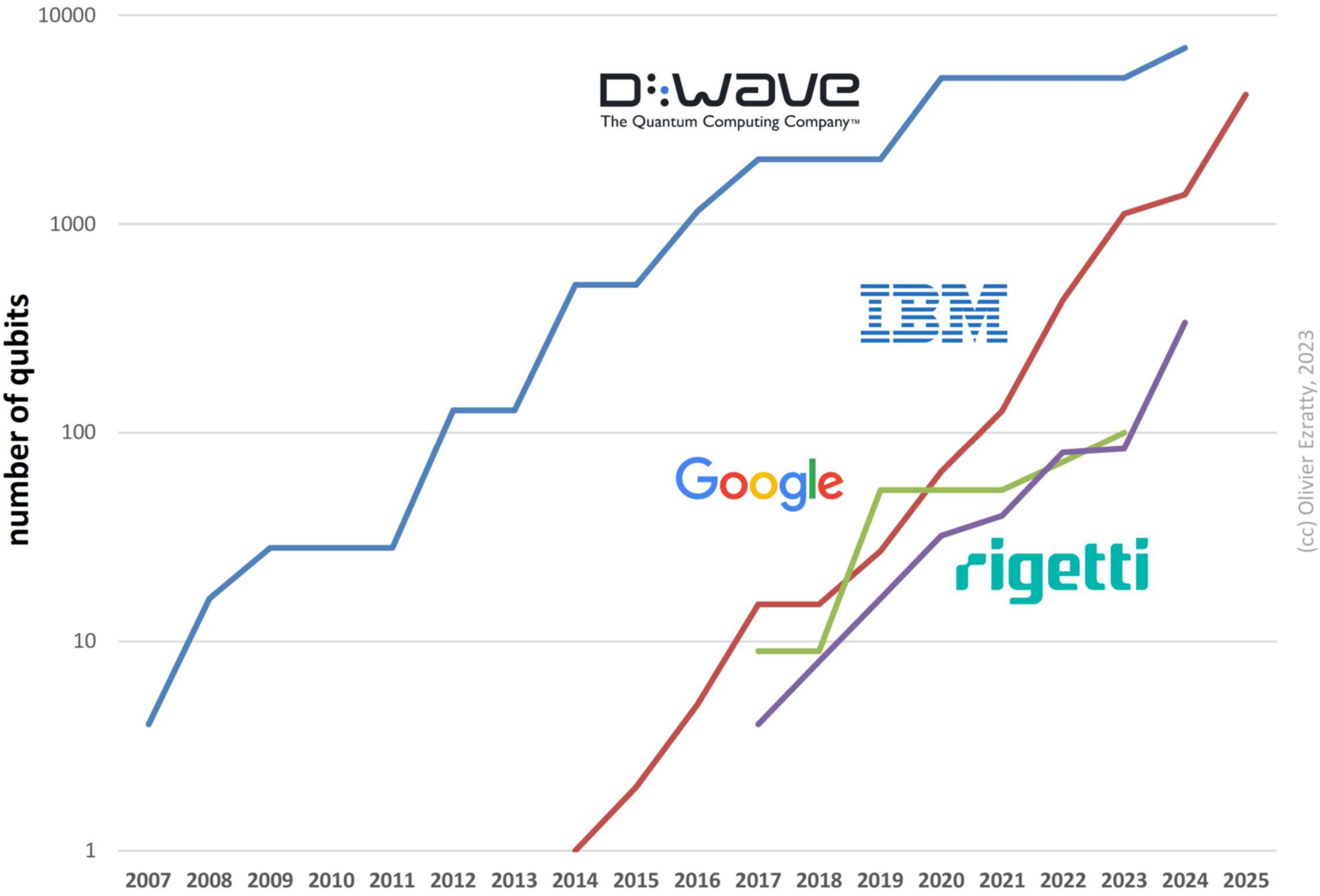


7x7
"3 errors at a time"
97 qubits

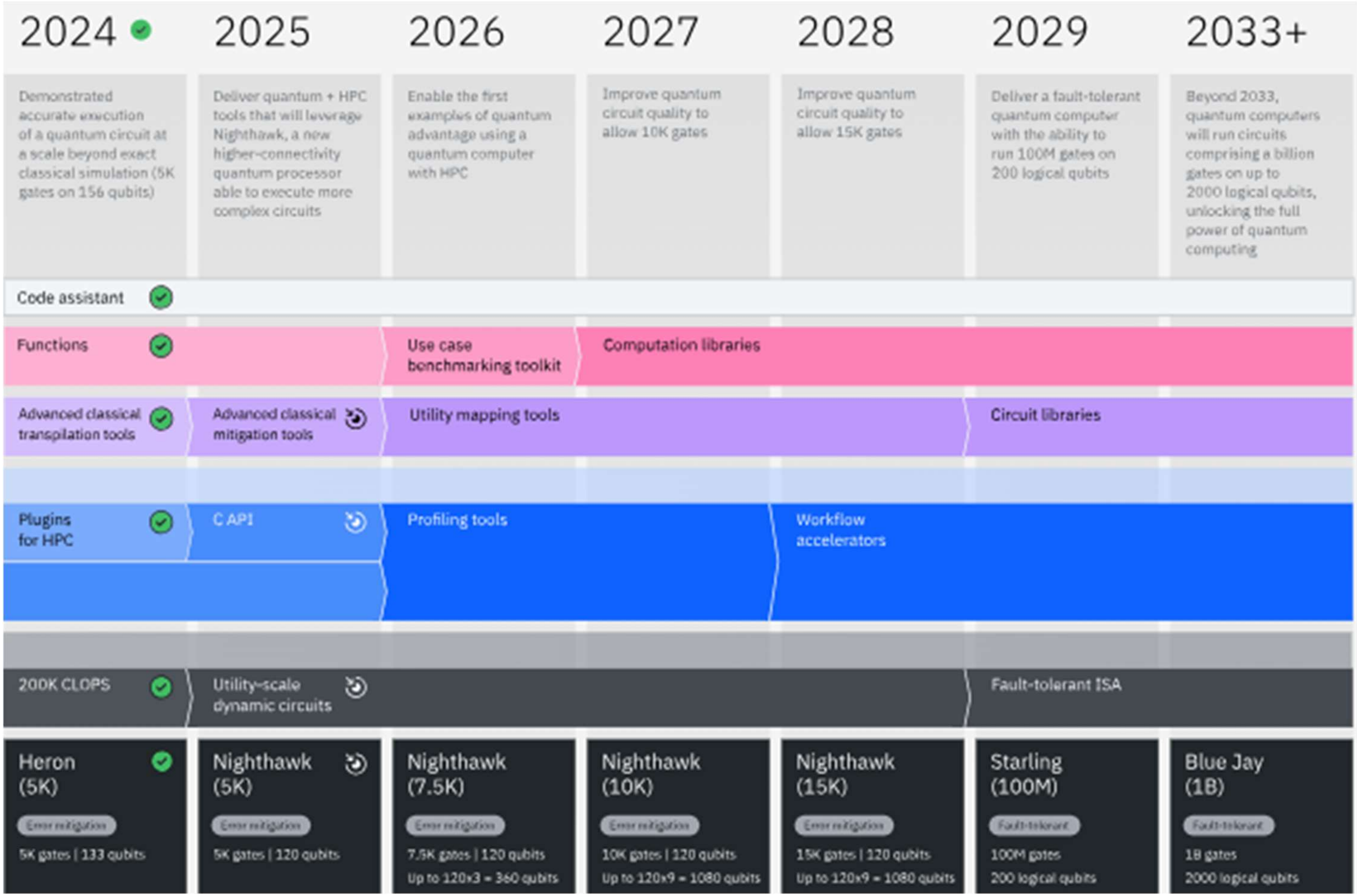
Article

Quantum error correction below the surface code threshold

“Quantum” Moore’s law?



Cf. IBM's roadmap



Thanks!