IIB行列模型における標準模型の創発

離散的手法による時空と場のダイナミクス

於　神戸理研

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No SUSY in nature

String vacua without space-time SUSY

One of the most important experimental discoveries of the past decade:

There is no SUSY in low energy ($< 2\mathrm{TeV}$).

The fact that has been known in string theory for more than 35 years:

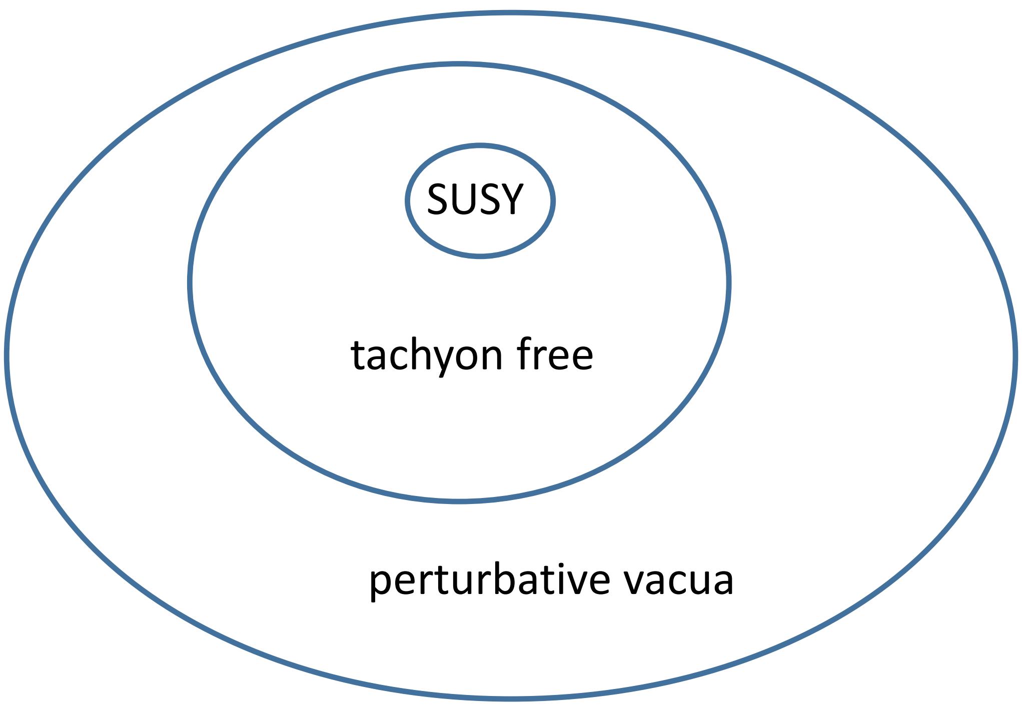
# of 4D tachyon free vacua without SUSY
>> # of 4D vacua with SUSY

Nevertheless, for some reason, low energy SUSY has been assumed and little consideration has been given to vacua without SUSY.

It is important not to confuse the existence of space-time SUSY with the non-existence of tachyons.

Space-time SUSY ⇒ No tachyons. true

No tachyons ⇒ Space-time SUSY false



Heterotic string

Space-time SUSY : tachyon free non SUSY

10D space-time

2 : 1

4D space-time

<<

(randomly generated)

HK-Lewellen-Tye, Dienes,

We cannot classify all possible perturbative vacua with 4D space-time.

But we can randomly construct 4D tachyon free vacua by free fermionic construction or orbifold.

Then we observe

# of vacua with space-time SUSY << that without SUSY.

Note Because of the modular invariance, absence of tachyon indicates that asymptotically

# of bosonic states $\sim$ # of fermionic states .

In other words,

tachyon free $\Leftrightarrow$ SUSY at Planck scale .

It is natural to expect that SUSY is broken at Planck scale.

Many perturbative vacua

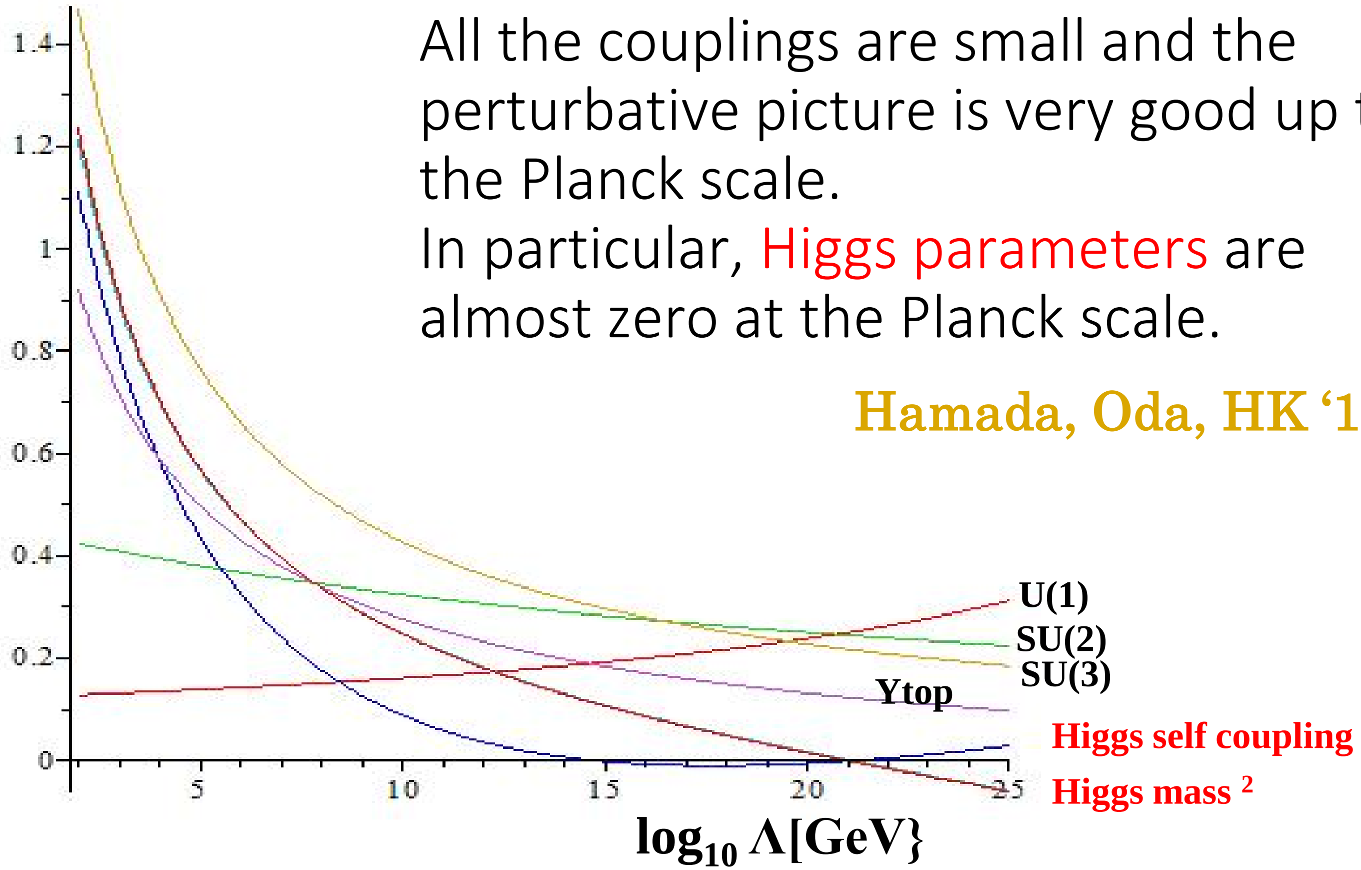
Basic question:
Is the degeneracy of the perturbative vacua resolved, when we consider non-pertutbative dynamics?

If the SUSY remains, the answer is probably no because the degeneracy of vacua is protected by SUSY. So we may still have a complicated "landscape" even after taking account of non-perturbative dynamics.

Fortunately, there is no SUSY in our nature. Therefore, nothing prevents the system from leaving the degenerate vacua and finding the true vacuum.

Possibility of Desert

RG analysis of SM

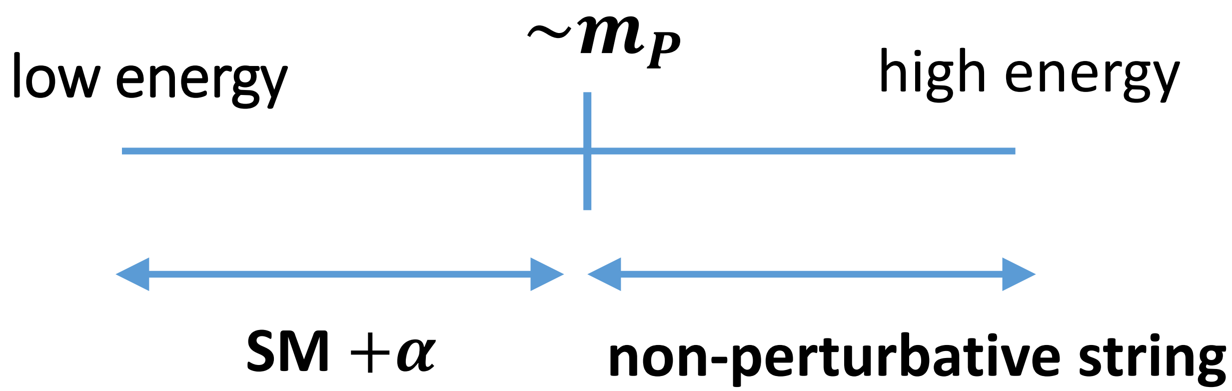


All the couplings are small and the perturbative picture is very good up to the Planck scale.
In particular, Higgs parameters are almost zero at the Planck scale.

Hamada, Oda, HK '13

It is natural to imagine that SM is directly connected to the string theory at the Planck scale without large modification.

possible scenario



i) The 4D space-time with SM+α field content emerges from the dynamics above the Planck scale of the non-perturbative string theory.

ii) The parameters of the field theory below the Planck scale should be determined from both stringy and low-energy dynamics.

PART 1
Matrix model and multiverse

IIB matrix model

Ishibashi, HK, Kitazawa, Tsuchiya '97

One candidate for a non-perturbative definition of string theory is the IIB matrix model

$$S_{\mathrm{IIB}} = -\mathrm{tr}\left(\left[A_\mu, A_\nu\right]^2\right) + \mathrm{tr}\left(\bar{\psi}\gamma^\mu\left[A_\mu, \psi\right]\right).$$

A_μ : 10D vector (10 bosonic components)
ψ : 10D M-W spinor (16 fermionic comp.)

Each component is an $N \times N$ Hermitian matrix.
We take $N \to \infty$ limit.

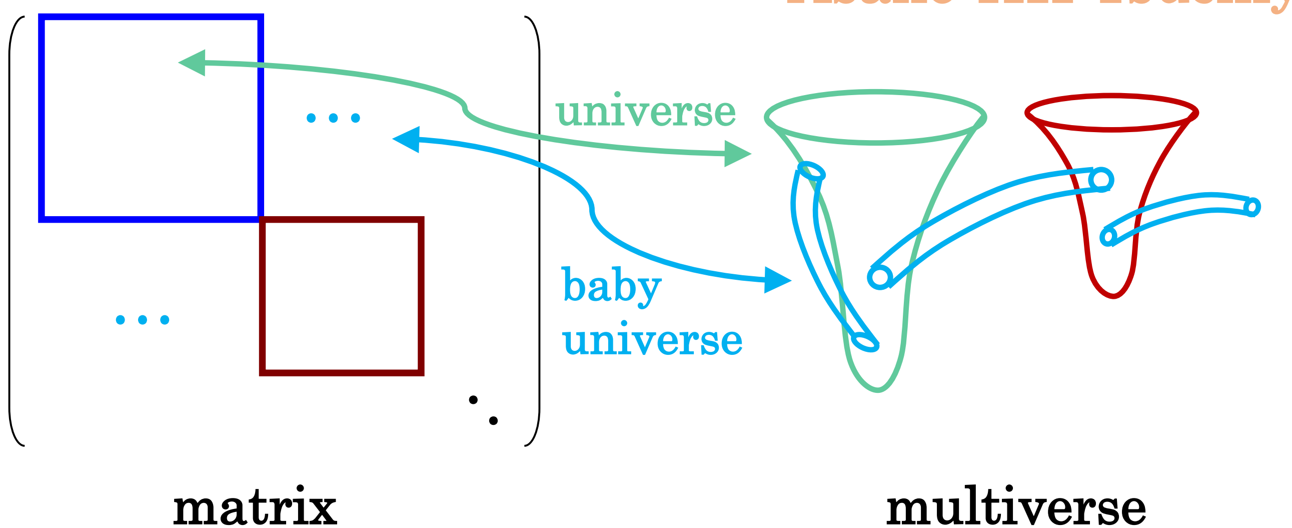
For a precise explanation, see YouTube
Hikaru Kawai "Emergence of space-time from matrices"
Institut des Hautes Études Scientifiques

multiverse and baby universes in MM

In IIB MM, universes arise from matrices.
By considering block diagonal configurations,
the multiverse emerges naturally.

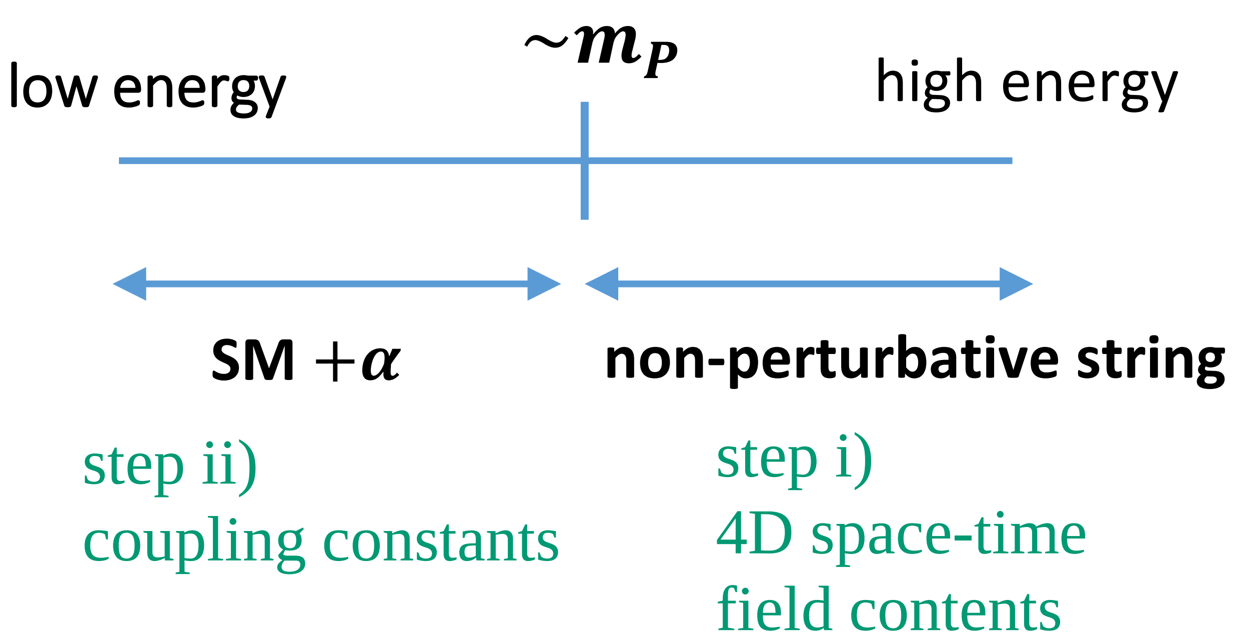
Integrating the fluctuations around the multiverse produces baby universes that connect two points in the multiverse, such as a space-time wormhole.

Asano-HK-Tsuchiya



In terms of the matrix model, the multiverse is the most natural starting point for discussing physics.

Perhaps the effect of the multiverse on step i) is not large, but the effect on step ii) can be large.



In fact, if the low energy effective theory is the ordinary QFT, it has the naturalness problem on the cosmological constant, Higgs mass and θ-parameter.

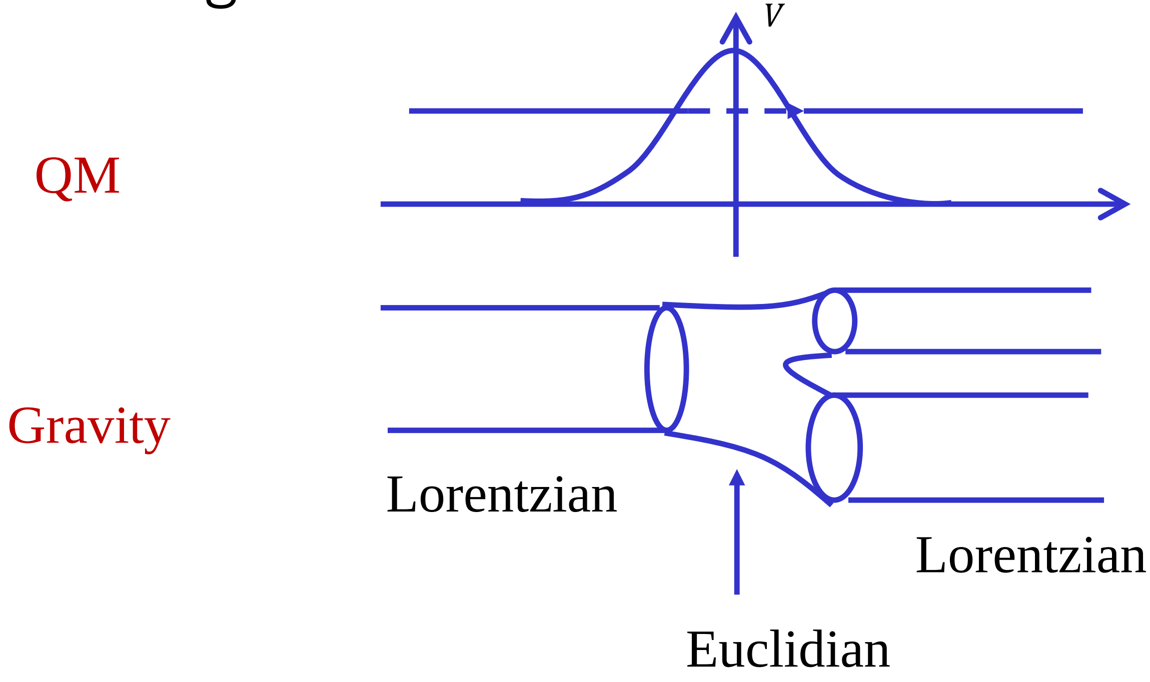
On the other hand it is possible that the low energy effective theory of multiverse is not the ordinary QFT but a generalized QFT that does not have the naturalness problem.

Low energy effective theory of multiverse

topology change, multiverse and baby universes

We want to consider the low energy effective theory of the multiverse.

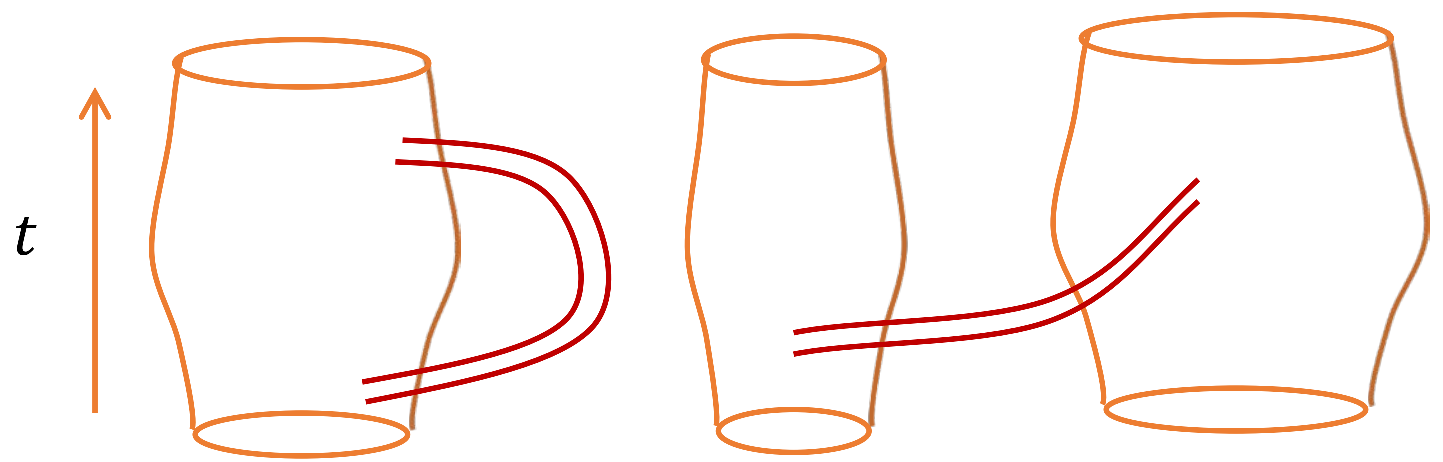
In terms of the WKB approximation of GR, topology change can be described by connecting Lorentzian and Euclidian signature.



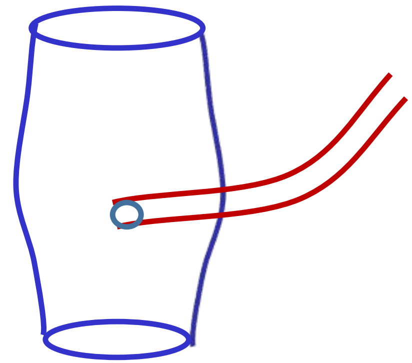
For macroscopic universes topology change is highly suppressed, because the transition probability is proportional to **exp(−classical Euclidean action).**

On the other hand, if one of the universes is small (baby universe), topology change becomes important.

What should be considered in low energy is emission and absorption of BU's by multiverse.



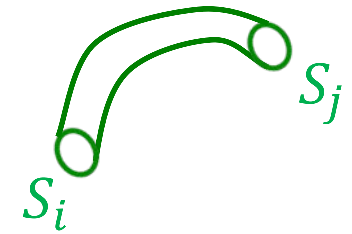
Low energy effective theory



From the large universe, emission or absorption of BU looks like the insertion of a local operator.

Therefore, the emission and subsequent absorption of a BU modify the effective action as

$$S \to S + \sum_{i,j} c_{ij} S_i S_j \; ,$$



where S_i is a space-time integral

$$S_i = \int d^4x \sqrt{-g(x)}\, O_i(x)$$

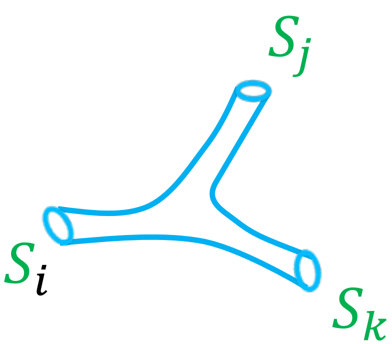
of a scalar operator O_i such as

$$1 \, , R \, , R_{\mu\nu} R^{\mu\nu} \, , F_{\mu\nu} \, F^{\mu\nu} , \overline{\psi} \gamma^{\mu} D_{\mu} \psi \, , \cdots .$$

Each S_i has a form of local action.

Furthermore, bifurcated BU's contribute to the effective action as

$$S \to S + \sum_{i,j,k} c_{ijk} S_i S_j S_k \ .$$



Thus the low energy effective theory of QG / MM is not a simple local action but a generalized QFT:

$$S_{\text{eff}} = f(S_1, S_2, \cdots).$$

Coleman '89
Asano-HK-Tsuchiya

Nature does fine tuning by itself.

Hamada, Kawana, HK   arXiv: 2210.05134
Kawana, HK, Oda, Yagyu   arXiv:2307.11420

Generalized QFT (or QM)?

Nambu, ...

ordinary QFT

$$\langle t_2, q_2 | t_1, q_1 \rangle = \int_{\substack{\mathrm{q}(t_2) = q_2 \\ \mathrm{q}(t_1) = q_1}} \mathcal{D}q \; e^{\frac{i}{\hbar} S[q]}$$

generalized QFT

$$\int \mathcal{D}q \, f(S_i[q]) \qquad S_i = \int d^4x \sqrt{-g(x)}\, O_i(x)$$

We will see that under some circumstances they are equivalent, and that the naturalness problem is resolved in the generalized QFT.

By using the Fourier transform of f

$$f(x) = \int \prod_i d\alpha_i \, w(\alpha) \, e^{i \sum_i \alpha_i x_i} \,,$$

the partition function of the generalized QFT is given by

$$Z = \int \mathcal{D}q \, f(S_i[q]) \qquad\qquad S_i = \int d^4x \sqrt{-g(x)} \, O_i(x)$$

$$= \int \mathcal{D}q \int \prod_i d\alpha_i \, w\,(\alpha) \, e^{i \sum_i \alpha_i S_i[q]}$$

$$= \int \prod_i d\alpha_i \, w(\alpha) Z(\alpha) \,,$$

where

$$Z(\alpha) = \int \mathcal{D}q \, e^{i \sum_i \alpha_i S_i[q]}.$$

Ordinary FT with
coupling constants α_i .

Generalized QFT = "superposition" of ordinary QFT

$$Z = \int \prod_i d\alpha_i \, w(\alpha) Z(\alpha)$$

Basic question:

If the integral is dominated by an infinitesimally small region around $\alpha_i = \alpha_i^{(0)}$, the theory is equivalent to the ordinary QFT with coupling constants $\alpha_i^{(0)}$.

The coupling constants are automatically tuned to

$$\alpha_i = \alpha_i^{(0)}.$$

Static vacuum

Static vacuum

We first assume that static vacua dominate in

$$Z = \int \prod_i d\alpha_i \, w(\alpha) Z(\alpha) \, .$$

In this case, we can regard

$$Z(\alpha) = e^{-i\, VF(\alpha)} \, ,$$

where V is the space-time volume and $F(\alpha)$ is the vacuum energy density.

The crucial point is that V appears only in the exponent

$$Z = \int d\alpha \, w(\alpha) \exp\bigl(-iVF(\alpha)\bigr) \, .$$

Therefor, when V is large, the integral is dominated by the extrema or singularities of $F(\alpha)$ as long as the function $w(\alpha)$ is smooth.

Because

singularity of $F(\alpha)$ = quantum phase transition,

we can say

"The coupling constants are automatically tuned to either a critical point or an extremum of the vacuum energy density."

Expanding universe

Time evolution of universe

If we consider the time evolution of universe, the notion of critical point should be generalized to critical point of the history of universe, which means coupling constants that significantly change the time evolution of universe when they are changed.

Generalized Multicritical Point Principle (GMPP):

The coupling constants of the low energy effective FT are automatically adjusted either to an extremum of the vacuum energy density or to a critical point of the history of universe.

Examples

1. QCD θ-parameter

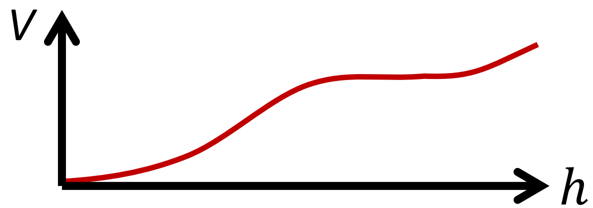
   $\theta = 0$ minimizes the vacuum energy.

2. Cosmological constant

   $\lambda = 0$ is a critical point of the history of universe.

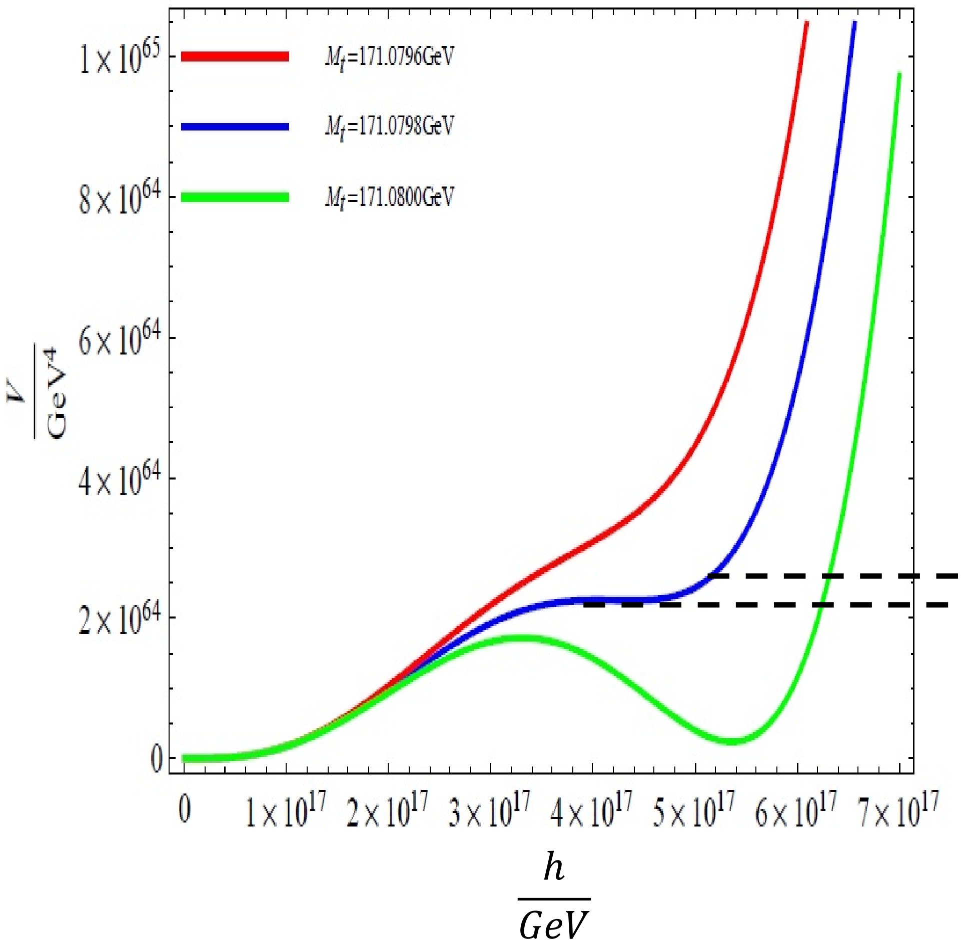
3. Higgs inflation at criticality

   Flat potential is a critical point of the history of universe.



Some (or all) of the parameters of SM$+\alpha$ are fixed by GMPP, independent of the detailed dynamics.

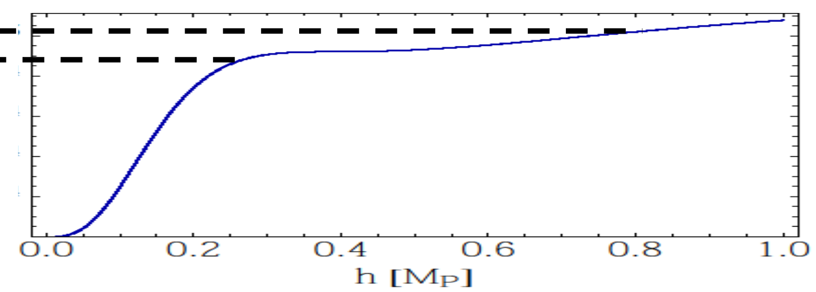
SM Higgs is close to MPP.



non-renormalizable coupling $\xi R h^2$ with $\xi \sim 10$.

In the Einstein frame the effective potential becomes

$$V(\varphi_h), \quad \varphi_h = \frac{h}{\sqrt{1+\xi h^2/M_P{}^2}}.$$



Hamada, Oda, Park and HK '14
Bezrukov,Shaposhnikov

How to explain non-zero m_H and CC.

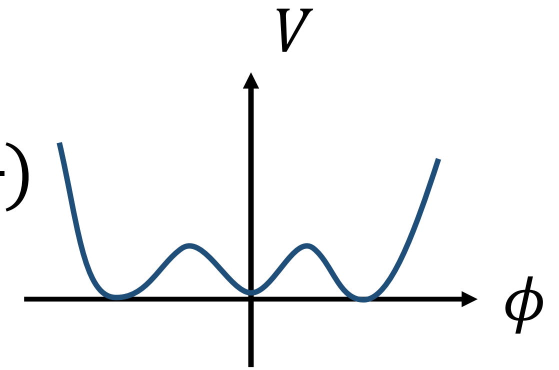
Possible solutions

<u>Higgs mass</u>

Coleman-Weinberg $m_H = m_P \exp(-\frac{c}{g^2})$

(1) 1 complex scalar + U(1) gauge

(2) 2 real scalars

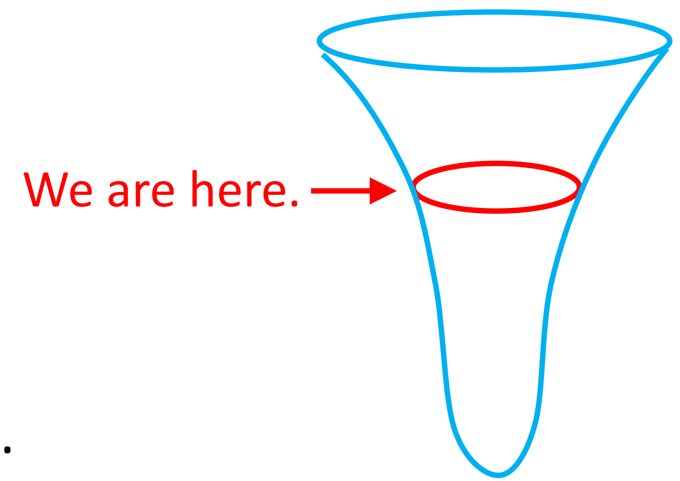


critical point

<u>Cosmological constant</u>

If we are at the moment of transition from a decelerating universe to an accelerating universe, then the current state of the universe is on the critical point.

$\Rightarrow$ vacuum energy$\sim (2\sim 3\ \mathrm{meV})^4$,
answer to the why-now problem ?



Summary and Conclusion for PART 1

Next possible direction of physics *on Nature*:

Ultimate goal:

Find the multiverse in constructive string theory and confirm that low-energy physics is described by the SM $+\alpha$ + EH action with the right coupling constants.

Divide the difficulties:

(1) Phenomenological approach to modification of SM.
Dark matter, Origin of inflation, ..
Key: Desert. Fine tuning by nature itself.

(2) Numerical or analytical investigation on emergent space-time and field.

(3) Further develop GMPP to determine low energy parameters.

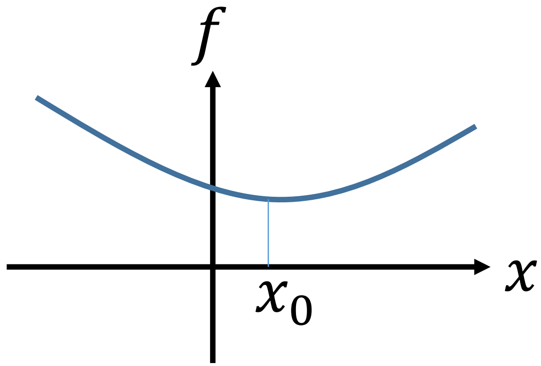
Appendix for PART 1

A. Formulas on $e^{i\,Vf(x)}$ for large V

$e^{iVf(x)}$ for large V

$f(x)$: real function

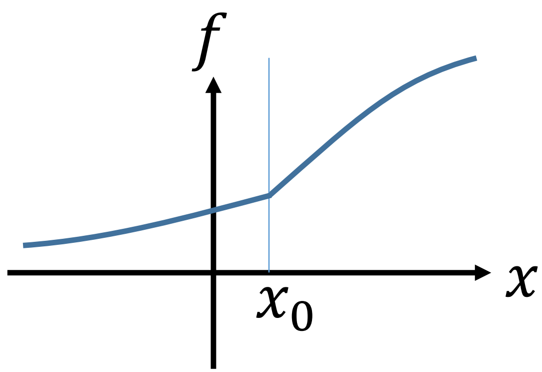
(1)

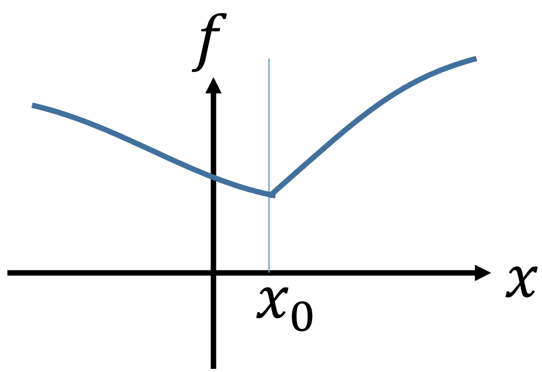


smooth and one extremum

$$e^{iVf(x)}$$
$$\sim \frac{1}{\sqrt{V}}\, e^{i\,Vf(x_0)}\delta(x-x_0)$$

(2)





f is continuous

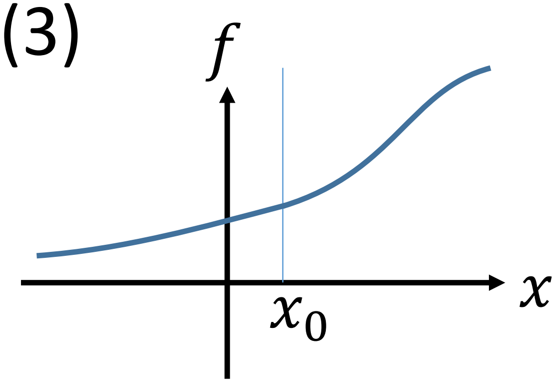
f' is discontinuous at x_0

x_0 need not be extremum

monotonic on each side $x \lessgtr x_0$

$$e^{iVf(x)}$$
$$\sim \frac{1}{V}\left(\frac{1}{f'(x_0+0)}-\frac{1}{f'(x_0-0)}\right)$$
$$\cdot\, e^{i\,Vf(x_0)}\,\delta(x-x_0)$$

(3)

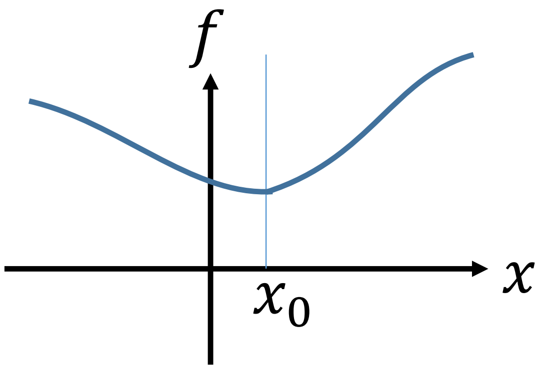


$f,\ f'$ are continuous

f'' is discontinuous at x_0

x_0 need not be extremum

monotonic on each side $x \lessgtr x_0$



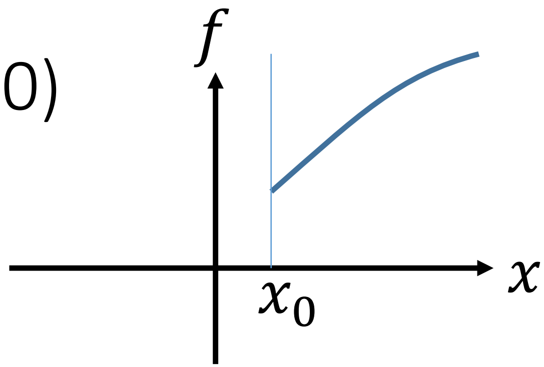
$$e^{iVf(x)}$$

$$\sim \frac{1}{V^2} \frac{1}{f'(x_0)^3} \left(f''(x_0+0) - f''(x_0-0)\right)$$

$$\cdot\, e^{i\,Vf(x_0)}\, \delta(x - x_0)$$

proof of (2), (3)

(0)



$f : [x_0, \infty] \to \mathbb{R}$
smooth
monotonically increasing

For any test function,
$\varphi \in C^\infty$, finite support,

$$\int_{x_0}^{\infty} dx \; e^{iVf(x)} \varphi(x)$$

$$= \int_{y_0}^{y_\infty} dy \frac{dx}{dy} \; e^{iVy} \varphi\left(f^{-1}(y)\right)$$

$\leftarrow y = f(x), y_0 = f(x_0), y_\infty = f(\infty)$

$$= \int_{y_0}^{y_\infty} dy \, e^{iVy} g(y)$$

$\leftarrow g(y) = \frac{dx}{dy} \varphi(f^{-1}(y))$

$$= \left[\frac{1}{iV} e^{iVy} g(y)\right]_{y_0}^{y_\infty} - \int_{y_0}^{\infty} dy \frac{1}{iV} e^{iVy} g'(y)$$

$$= \frac{i}{V} e^{iVy_0} g(y_0) + O\left(\frac{1}{V^2}\right)$$

$\leftarrow \varphi$ has finite support $\Rightarrow g(y_\infty) = 0$

$$= \frac{i}{V} e^{iVf(x_0)} \frac{1}{f'(x_0)} \; \varphi(x_0) + O\left(\frac{1}{V^2}\right)$$

B. Creation and annihilation of universe

Hamiltonian for the universe

The space is assumed to be S^3.

temporal gauge: $ds^2 = -dt^2 + \gamma_{ij}\,(t,x) dx^i dx^j$

x^i: coordinates of S^3

mini-superspace: $ds^2 = -dt^2 + a(t)^2 d\Omega^2$

$d\Omega^2$: metric of unit S^3

Hamiltonian derived from EH action is

$m_P = 1$

$$H_{\text{uiverse}} = -\frac{p_a^2}{2\,a} + \frac{a^3}{6}\lambda(a),\ \ p_a = a\,\dot{a}\,.$$

←wrong sign

$$\lambda(a) = -\frac{1}{a^2} + \rho(a)$$

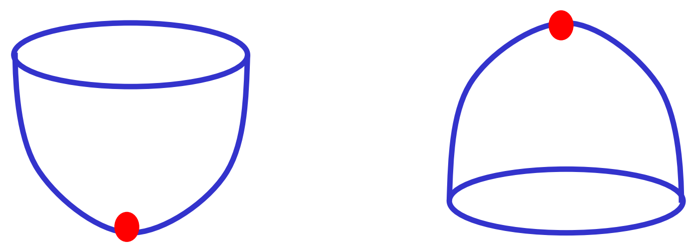
$$\rho(a) = \lambda + \varepsilon(a)$$

λ: cosmological constant

ε: energy density of matter and gravitons

creation and annihilation of universe

EH action does not tell anything about creation and annihilation of the infinitesimally small universe. Once it is created its time evolution is described by the Hamiltonian before it is annihilated.



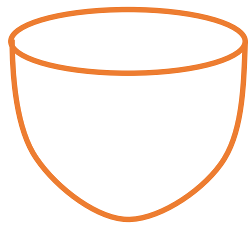
This is a special case of topology change. But in this case we may be able to introduce the transition amplitude by hand.

At any rate, the ultimate answer for topology change would be obtained by an constructive definition of string such as IIB MM.

Creation of universe from nothing and BU

For simplicity we assume all universes have S^3 topology.

Then the creation of small universe from nothing can be described by the mini super space Hamiltonian:



$$H = -\frac{p^2}{2a} - a + \lambda a^3 + \frac{C}{a}$$

$-a$: curvature term

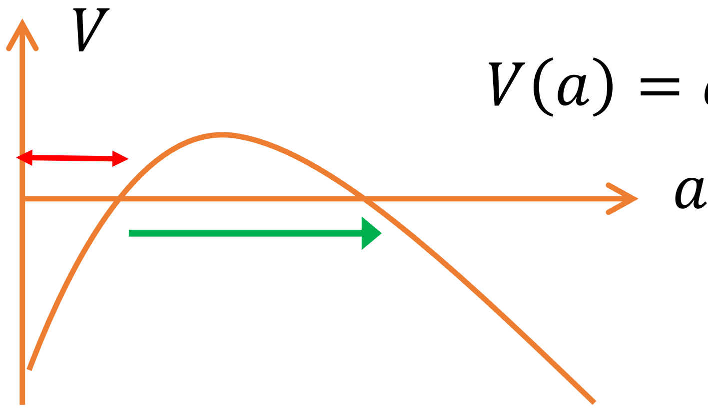
λ : cosmological constant at short distances

C/a: energy of the fields (radiation like)

Then the potential for a is given by

$$-H = \frac{p^2}{2a} + V(a), \ V(a) = a - \lambda a^3 - \frac{C}{a}.$$

If $\lambda \sim C \sim 1$ (Planck unit), the potential looks like



$$V(a) = a - \lambda a^3 - \frac{C}{a}$$

If λ is small and positive, there is a potential barrier for the universe to pop out from nothing ($a = 0$).

Furthermore, if $C > 0$, a real (not virtual) state exists near $a \sim 0$, which can be regarded as a baby universe.

Baby universe can tunnel to a "real universe" and then grow to a large universe.

PART 2
Standard Model from IIB Matrix Model

Based on collaborations with

(1) P.-M. Ho, W. Piensuk, and W.-H. Shao , to appear,

(2) P.-M. Ho, H . C. Steinacker, e-Print: 2509.06646.

Matrices as bi-local fields

1. IIB matrix model

Ishibashi-HK-Kitazawa-Tsuchiya

$$S_{\mathrm{IIB}} = \alpha \, Tr\left(-\frac{1}{4}\left[A_\mu, A_\nu\right]^2 - \frac{1}{2}\bar{\psi}\Gamma^\mu\left[A_\mu, \psi\right]\right)$$

A_μ and ψ are Hermitian operators on a vector space V.

$V = \mathbb{C}^N$

$\Rightarrow$ A_μ and ψ are $N \times N$ matrices

$(A_\mu)_{ij}\,,(\psi_s)_{ij}$

$V = \{\, \varphi(x): \mathbb{R}^d \to \mathbb{C}\}$ the space of functions on $\mathbb{R}^d$

$\Rightarrow$ A_μ and ψ are bilocal fields

$$A_\mu(x,y) = \langle x|A_\mu|y\rangle\,,\ \ \psi_s(x.y) = \langle x|\psi_s|y\rangle$$

In the case $V = \{\, \varphi(x)\colon \mathbb{R}^d \to \mathbb{C}\}$, the action becomes

$$
\begin{aligned}
S = -\frac{\alpha}{2}\int d^d x d^d y d^d z d^d w \\
&\{A_\mu(x,y)A_\nu(y,z)A_\mu(z,w)A_\nu(w,x) \\
&-A_\mu(x,y)A_\nu(y,z)A_\nu(z,w)A_\mu(w,x)\} \\
+\int d^d x d^d y d^d z \\
&\{\bar{\psi}(x,y)\Gamma^\mu A_\mu(y,z)\psi(z,x) \\
&-\bar{\psi}(x,y)\Gamma^\mu \psi(y,z)A_\mu(z,x)\}.
\end{aligned}
$$

Diffeomorphism invariance

This is invariant under the diffeomorphisms on $\mathbb{R}^d$:

$$A_\mu(x,y) \mapsto J(x)^{\frac{1}{2}} J(y)^{\frac{1}{2}} A_\mu(\varphi(x),\varphi(y)),$$

$$\psi(x,y) \mapsto J(x)^{\frac{1}{2}} J(y)^{\frac{1}{2}} \psi(\varphi(x),\varphi(y)),$$

where $J(x) = \det(\partial_\alpha \varphi^\beta(x))$.

It is because

$$\begin{aligned} &\int d^d y \cdots \phi(x,y)\phi(y,z)\cdots \\ &\mapsto \int d^d y \cdots \phi\big(\varphi(x),\varphi(y)\big)\, J(y)\, \phi\big(\varphi(y),\varphi(z)\big)\cdots \\ &\quad = \int d^d y' \cdots \phi(x',y')\phi(y',z')\cdots. \end{aligned}$$

This a small subset of the $\mathrm{SU}(N)$ invariance:

$$|x\rangle \mapsto J(x)^{\frac{1}{2}}\, |\varphi(x)\rangle \quad \sim \quad \text{permutation of } |i\rangle$$

IIB matrix model as pre-geometric theory

We can view this action as a pre-geometric theory, i.e., a theory that, when a certain background is introduced, becomes an ordinary bi-local field on curved spacetime.

It is natural to expect that the low-energy effective theory of this action is a field theory involving gravity.

Roughly speaking, this is true, but we have to be careful about unitarity (or positivity).

This is because the vector and spinor indices of A and ψ are like Lorentz indices which are not transformed under the diffeomorphisms.

$$A_\mu(x,y) \mapsto J(x)^{\frac{1}{2}} J(y)^{\frac{1}{2}} A_\mu(\varphi(x),\varphi(y))$$

$$\psi_s(x,y) \mapsto J(x)^{\frac{1}{2}} J(y)^{\frac{1}{2}} \psi_s(\varphi(x),\varphi(y))$$

However, the IIB matrix model has no built-in local Lorentz invariance.

Therefore the situation is similar to an action written in terms of the vierbein $e_a^{\ \nu}$ but without local Lorentz symmetry.

In general, such theory is not unitary because the negative norm states $e_a^{\ 0}$ cannot be removed.

Constraints to guarantee unitarity

In fact, as we will see, one simple way to guarantee unitarity is to impose the following constraints on the bilocal fields.

$$\int d^d y \, A_\mu(x,y) = 0, \quad \int d^d y \, \psi_s(x,y) = 0 \, .$$

If we do so, the symmetry is reduced from the diffeomorphisms to the volume preserving diffeomorphisms (VPDs)

$$J(x) = \det\left(\partial_\alpha \varphi^\beta(x)\right) = 1 \, ,$$

because these constraints are invariant under the diffeomorphisms only when $J = 1$.

Meaning of the constraints

$\leftarrow \phi(x,y) = \langle x|\phi|y\rangle$

$$\int d^d y\, \phi(x,y) = 0 \;\Leftrightarrow\; \phi|k=0\rangle = 0$$

Here $|k\rangle$ is the momentum eigenstates:

$$|k\rangle = \int d^d x \exp(i\, k\cdot x)\, |x\rangle$$

$$\Rightarrow \; |k=0\rangle = \int d^d y\, |y\rangle\,.$$

These constraints correspond to remove $|k=0\rangle$ from the Hilbert space $V = \{\varphi : \mathbb{R}^d \to \mathbb{C}\}$.

In other words, we replace

$$V = \langle |k\rangle \rangle_{k\in\mathbb{R}^d} \;\text{ with }\; V' = \langle |k\rangle \rangle_{k\in\mathbb{R}^d - \{0\}}\,.$$

This change seems very minor, but it imposes some nontrivial constraints.

formal Taylor expansion with respect to $\hat{p}_\mu$
= differential operator

$$\phi = a(\hat{x}) + \tfrac{1}{2}\left(\hat{p}_\mu a^\mu(\hat{x}) + a^\mu(\hat{x})\hat{p}_\mu\right)$$
$$+ \frac{1}{2}\left(\hat{p}_\mu \hat{p}_\nu a^{\mu\nu}(\hat{x}) + a^{\mu\nu}(\hat{x})\hat{p}_\mu \hat{p}_\nu\right) + \cdots$$
$$= a(x) + \tfrac{i}{2}\left(\partial_\mu a^\mu(x) + a^\mu(x)\partial_\mu\ \right)$$
$$+ \frac{i^2}{2}\left(\partial_\mu \partial_\nu a^{\mu\nu}(x) + a^{\mu\nu}(x)\partial_\mu \partial_\nu\right) + \cdots$$

$$\phi\ |k = 0\rangle = 0 \ \Leftrightarrow$$
$$\begin{cases} a(x) - \tfrac{1}{2}\ \partial_\mu \partial_\nu a^{\mu\nu}(x) + \cdots = 0, \\ \ \partial_\mu a^\mu(x) - \ \partial_\mu \partial_\nu \partial_\lambda a^{\mu\nu\lambda}(x) + \cdots = 0\,. \end{cases}$$

More precisely,
suppose we expand A_μ and ψ_s as differential operators

$$A_\mu = a_\mu(x) + \frac{i}{2}\left(\partial_\nu a_\mu^{\ \nu}(x) + a_\mu^{\ \nu}(x)\partial_\nu\ \right) + \frac{i^2}{2}\left(\partial_\nu \partial_\lambda a_\mu^{\ \nu\lambda}(x) + a_\mu^{\ \nu\lambda}(x)\partial_\nu\partial_\lambda\right) + \cdots$$

From physical intuition, we expect that the higher rank tensors become massive while the lower rank tensors remain massless because of the diffeomorphism symmetry.

Then, we need gauge symmetry or some constraints to eliminate negative norm states for massless fields.

We will show that the constraints indeed fulfill this role.

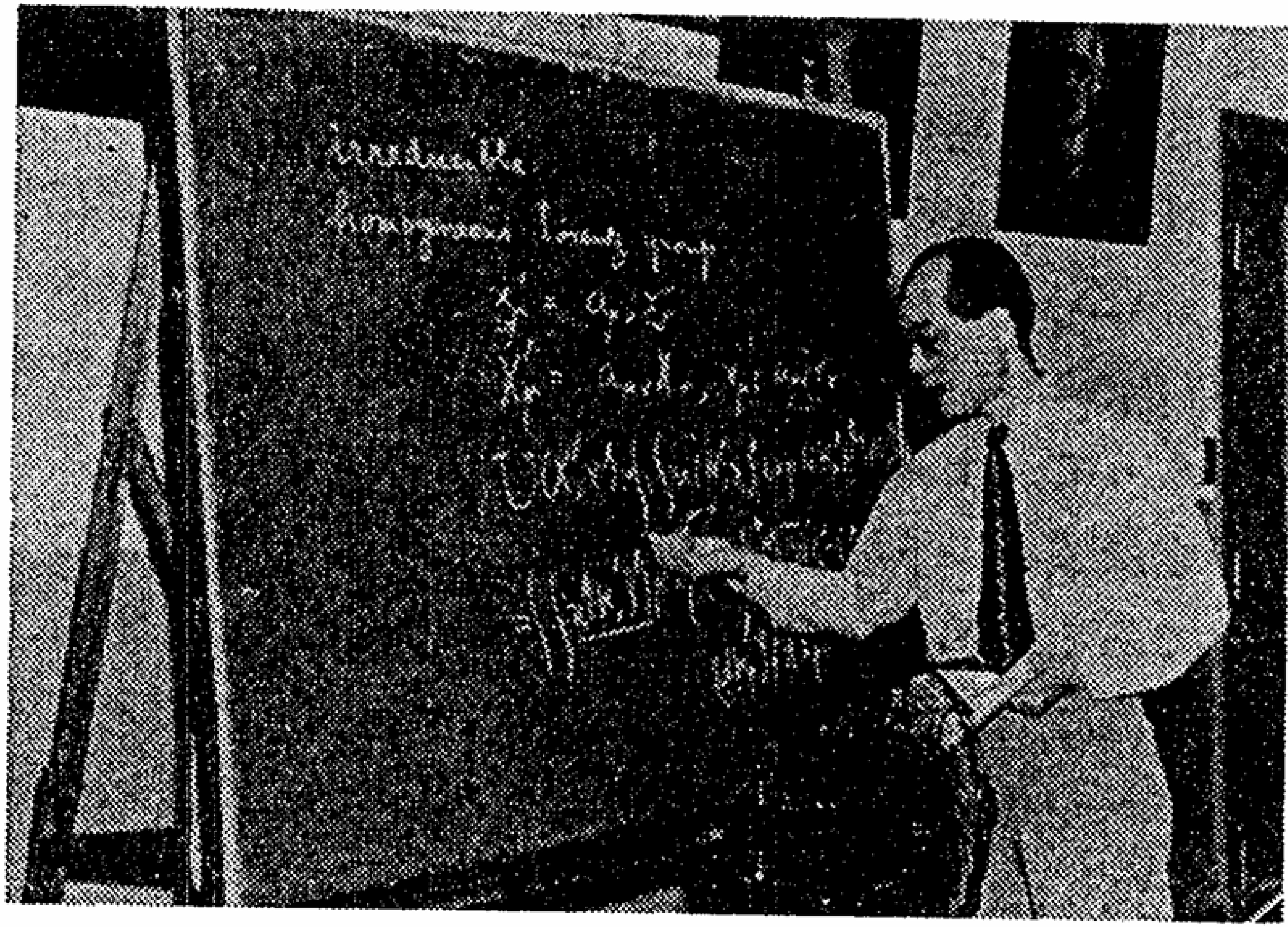
Yukawa and bi-local field

For some reason, Yukawa did not like renormalization theory and instead tried to understand elementary particles by extended fundamental object.

He tried many kinds of nonlocal fields.
Bilocal field, Tetralocal field, .. , Elementary domain

Phys, Rev. 77 (1950) 219, 80 (1950) 1047, …

$$\left(\frac{\partial^2}{\partial X^{\mu 2}} - m^2\right)\psi(X,\xi) = 0,$$
$$(\xi^2 - \lambda^2)\psi(X,\xi) = 0,$$
$$\xi^\mu \frac{\partial}{\partial X^\mu}\psi(X,\xi) = 0.$$



京大における講演 (1950)

JPS journal, vol 37, no.4 (1982), p.276　By O. Hara

Fluctuations around diagonal configurations

2. Commuting background

In the case $V = \mathbb{C}^N$, one of the simplest classical solutions is given by the diagonal matrices:

$$A_\mu^{(0)} = \begin{cases} P_\mu \ (\mu = 1 \sim d), \ \ \left(P_\mu\right)_{ij} = p_\mu^i \delta_{ij} \ \text{(diagonal)}, \\ 0 \ (\ \mu = d+1 \sim 10) \end{cases}$$

$$\psi^{(0)} = 0.$$

Let's translate this to bilocal fields.

Corresponding solution in $V = \{ \varphi(x) : \mathbb{R}^d \to \mathbb{C} \}$

Using the momentum representation

$$|k\rangle = \int d^d x \exp(ik \cdot x)|x\rangle,$$

we introduce the following identification:

$$|i\rangle \in \mathbb{C}^N \leftrightarrow \ |k = p^i\rangle \ .$$

Then the diagonal solution

$$\left\langle i \middle| A_\mu^{(0)} \middle| j \right\rangle = \begin{cases} p_\mu^i \ \delta_{ij} \ (\mu = 1 \sim d) \\ 0 \qquad\ \ (\mu = d + 1 \sim 10) \end{cases}, \qquad \langle i | \psi^{(0)} | j \rangle = 0$$

becomes

$$\left\langle k \middle| A_\mu^{(0)} \middle| k' \right\rangle = \begin{cases} k_\mu \delta(k - k') \ \ (\mu = 1 \sim d) \\ \quad 0 \qquad\qquad\quad (\mu = d + 1 \sim 10) \end{cases}, \ \langle k | \psi^{(0)} | k' \rangle = 0.$$

By introducing $\hat{x}^\mu$ and $\hat{p}_\mu$ as the ordinary QM

$$[\hat{x}^\mu, \hat{p}_\nu] = i\delta^\mu{}_\nu,$$

we can express

$$A_\mu^{(0)} = \begin{cases} \hat{p}_\mu \ (\mu = 1\sim d) \\ 0 \quad (\mu = d+1\sim 10) \end{cases}$$

$$\psi^{(0)} = 0\,.$$

In terms of the coordinate representation,

$$\left\langle x \middle| A_\mu^{(0)} \middle| x' \right\rangle = \begin{cases} -i\,\partial_\mu \delta(x - x'\,) & (\mu = 1\sim d) \\ \quad 0 & (\mu = d+1\sim 10) \end{cases}$$

$$\left\langle x \middle| \psi^{(0)} \middle| x' \right\rangle = 0.$$

$A_\mu^{(0)}$ = Flat spacetime background

Commutator with $A_\mu^{(0)}$ is the derivative with respect to the center of mass coordinate of the bilocal fields.

In fact,

$$\left\langle x \left| \left[A_\mu^{(0)}, \phi\right] \right| x' \right\rangle = \langle x | [\hat{p}_\mu, \phi] | x' \rangle = -i \left(\frac{\partial}{\partial x^\mu} + \frac{\partial}{\partial x'^\mu} \right) \langle x | \phi | x' \rangle.$$

$$= -i \frac{\partial}{\partial X^\mu} \phi \left(X + \frac{\xi}{2}, X - \frac{\xi}{2} \right),$$

where X and ξ are the CM and the relative coordinates

$$X = \frac{1}{2}(x + x'),\ \xi = x - x'.$$

We can regard $A_\mu^{(0)}$ as a flat space background of the pre-geometric action.

3. Quenched reduced model and bilocal field

Parisi, Gross-Kitazawa, Bhanot-Heller-Neuberger

If

(1) the diagonal elements of A_μ are quenched uniformly in $\mathbb{R}^d$ (with some cut off),

and

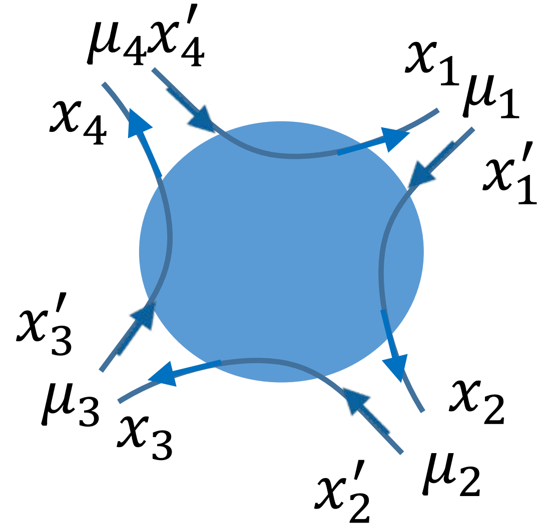
(2) only planar diagrams are considered,

then

the YM-type matrix model is equivalent to the corresponding d-dim YM theory.

More precisely, L pt. Green's functions of the MM and the d-dim YM are related as

$$\left\langle \langle x_1 | A_{\mu_1} | x_1' \rangle \cdots \langle x_L | A_{\mu_L} | x_L' \rangle \right\rangle_{\mathrm{MM}}$$
$$= \delta(\xi_1 + \cdots + \xi_L)\, G(X_1, \mu_1, \cdots, X_L, \mu_L)$$



$$\langle A^{a_1}_{\mu_1}(X_1) \cdots A^{a_L}_{\mu_L}(X_L) \rangle_{\mathrm{YM}}$$
$$= \mathrm{tr}(t^{a_1} \cdots t^{a_L})\, G(X_1, \mu_1, \cdots, X_L, \mu_L)$$

<u>correspondence</u>

$$\delta(\xi_1 + \cdots + \xi_L) = \mathrm{tr}(t^{a_1} \cdots t^{a_L})$$

Roughly speaking,

N^2 DOF of adjoint rep. $\Leftrightarrow$ continuous DOF of ξ

In particular, there is no negative norm state in the bi-local field representation of IIB MM:

Degeneracy of N^2 particles is resolved by the dynamics of diagonal elements.

For example, $\langle A_i \rangle \ (i = d + 1 \ldots 10) \quad SU(N) \ \rightarrow U(1)^N$

If gravity is there, it must arise from the collective motion of diagonal elements.

But its microscopic mechanism might be complicated.

However, as we shall see, the graviton does indeed appear as a type of Nambu-Goldstone particle.

Graviton as Nambu-Goldstone boson

5. Graviton and NG boson

Sometimes it is said that gravitons can be regarded as NG bosons.

e.g. P. R. Phillips, "Is the Graviton a Goldstone Boson?," Phys.Rev. 146 (1966) 966.

The argument is general, but for simplicity let's consider the Einstein-Hilbert action.

(1) The original action is diffeomorphism invariant.

(2) If we consider the harmonic gauge, the action is invariant under global $GL(d)$:

$$g_{\mu\nu} \to g'_{\mu\nu} = M_{\mu}{}^{\mu'} M_{\nu}{}^{\nu'} g_{\mu'\nu'}\ \text{, for } M_{\mu}^{\nu} \in GL(d).$$

(3) A typical vacuum is given by $g_{\mu\nu}(x) = \eta_{\mu\nu}$.

This vacuum breaks $GL(d)$ to $SO(d-1,1)$.

(4) The other vacua $g_{\mu\nu}(x) = \mathbf{const}$ are obtained from $\eta_{\mu\nu}$ by $GL(d)$.

(5) Therefore the vacuum moduli is given by

$GL(d)/SO(d-1,1) \cong$ rank 2 symmetric tensor ,

(6) We have corresponding NG bosons.

This is nothing but $h_{\mu\nu}$, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$.

6. VPD in MM and GR

(1) The original action is invariant under

$$SO(d-1,1)\otimes\mathrm{VPD}.$$

(2) If we consider a Lorenz type gauge $\left[A_\mu^{(0)}, A_\mu\right]=0$,

the action is invariant under global

$$SO(d-1,1)\otimes SL(d).$$

(3) We assume that a typical vacuum has a

translationally and rotationally invariant VEV of A_μ:

$$\left\langle\langle x|A_\mu|y\rangle\right\rangle=\frac{\partial}{\partial x^\mu}f\left((x-y)^2\right).$$

This vacuum breaks

$SO(d-1,1)\otimes SL(d)$ to diagonal $SO(d-1,1)$.

(4) We assume that the other vacua are obtained from

$$\left\langle \langle x|A_\mu|y\rangle \right\rangle = \frac{\partial}{\partial x^\mu} f\left((x-y)^2\right)$$

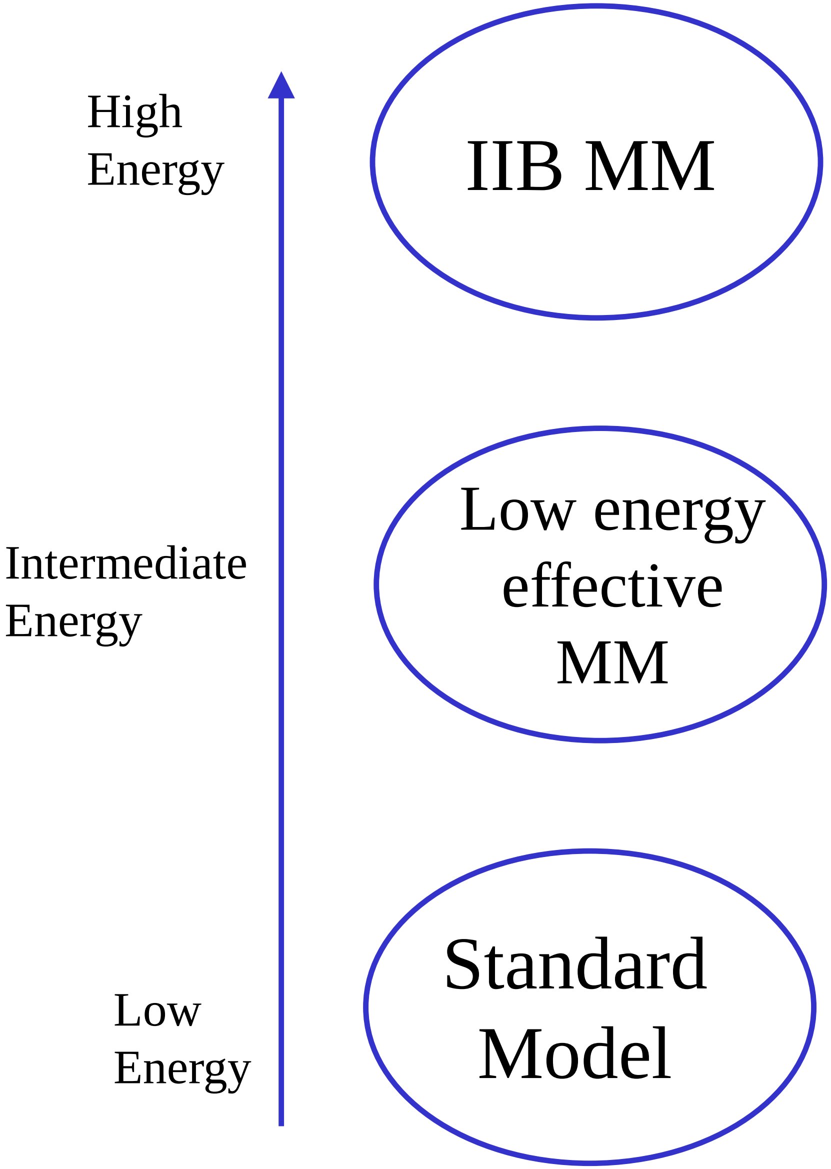
by $SO(d-1,1) \otimes SL(d)$.

(5) Then the vacuum moduli is given by

$$SO(d-1,1) \otimes SL(d) / SO(d-1,1) \cong SL(d)$$

(6) We have corresponding NG bosons that consist of traceless symmetric and anti-symmetric tensors.

Low energy effective theory



Working Hypnoses

Low energy behavior is described by a small IIB MM where matrices are regarded as elements of a Lie algebra.

7. Excitations around vacuum

Suppose the emergent space is a flat space, e.g. $\mathbb{R}^d$.

We express the bi-local fields as a formal power series of derivatives:

real

$$\langle x|A_\mu|y\rangle = \sum_{n=0}^{\infty} a_\mu^{(n)\,\nu_1\cdots\nu_n}(X)\, i^n \partial_{\nu_1}\cdots\partial_{\nu_n}\delta(\xi)\,,$$

$$\langle x|\Psi_s|y\rangle = \sum_{n=0}^{\infty} \psi_s^{(n)\,\nu_1\cdots\nu_n}(X)\, i^n \partial_{\nu_1}\cdots\partial_{\nu_n}\delta(\xi).$$

Physically, we expect only lower spin states are massless.

$a_\mu^{(0)}(x)$: gauge fields

${a^{(1)}}_\mu^{\ \nu}(x)$: vierbein

$\psi_s^{(0)}(x)$: quark-lepton fields

If we keep only two terms in A_μ , we have

$$\langle x|A_\mu|y\rangle = a_\mu(X)\delta(\xi) + ia_\mu{}^\nu(X)\partial_\nu\delta(\xi).$$

The condition for unitarity becomes

$$0 = \int dy\, \langle x|A_\mu|y\rangle = a_\mu(x) + i\partial_\nu a_\mu{}^\nu(x)$$

$$\Leftrightarrow \;\; a_\mu = 0, \quad \partial_\nu a_\mu{}^\nu = 0$$

In terms of differential operator,

$$A_\mu = i\, a_\mu{}^\nu(x)\partial_\nu\, , \partial_\nu a_\mu{}^\nu = 0$$

Each of A_μ is an infinitesimal VPD.

Minimal model

We can consider a simplified matrix model where the matrix algebra is replaced to the set of infinitesimal VPD's on $\mathbb{R}^d$.

$$\mathrm{End}(V) \implies \mathcal{S} = \{v^\mu \partial_\mu \,, \partial_\mu v^\mu = 0\}$$

Because it forms a Lie algebra, we can consider

EOM $\left[A_\nu, \left[A_\mu, A_\nu\right]\right] = 0$

Gauge transformation

$$\delta_v A_\mu = \left[v,\ A_\mu\right],\ v = v^\mu \partial_\mu \,, \partial_\mu v^\mu = 0$$

We can regard this as the low energy effective theory around the flat spacetime.

Unitarity of the minimal model

8. Weak field expansion

In order to check the unitarity of the minimal model, we start with weak field expansion:

$$A_\mu = a_\mu{}^\nu(x)\partial_\nu, \;\; a_\mu{}^\nu = \delta_\mu{}^\nu + c_\mu^\nu \, , \;\; \partial_\nu c_\mu^\nu = 0 \, .$$

Then the EOM $\left[A_\nu, [A_\mu, A_\nu]\right] = 0$ becomes

$$[A_\mu, \; A_\nu]^\rho = (\delta_\mu{}^\lambda + c_\mu^\lambda)\partial_\lambda(\delta_\nu{}^\rho + c_\nu^\rho) - (\mu \leftrightarrow \nu)$$
$$= \partial_\mu c_\nu^\rho - \partial_\nu c_\mu^\rho + O(c^2)$$
$$\left[A_\nu, [A_\mu, A_\nu]\right]^\lambda = \partial_\nu(\partial_\mu c_\nu^\lambda - \partial_\nu c_\mu^\lambda) + \mathrm{O}(\mathrm{c}^2) = 0$$

The gauge transformation becomes

$$\delta_v \, c_\mu^\lambda = [v, \; A_\mu]^\lambda = v^\nu\partial_\nu(\delta_\mu{}^\lambda + c_\mu^\lambda) - (\delta_\mu{}^\nu + c_\mu^\nu)\partial_\nu v^\lambda$$
$$= -\partial_\mu v^\lambda + O(c)$$

EOM $\partial_\nu\left(\partial_\mu c_\nu^{\lambda} - \partial_\nu c_\mu^{\lambda}\right) = 0\,,\ \partial_\nu c_\mu^{\nu} = 0\,.$

Gauge transformation $\delta_v\, c_\mu^{\lambda} = -\partial_\mu v^{\lambda}\,, \partial_\mu v^{\mu} = 0.$

Using the gauge symmetry, we can fix the gauge to Lorenz like gauge: $\partial_\nu c_\nu^{\lambda} = 0.$

Then we have

$$\square c_\mu^{\nu} = 0\,,\ \partial_\nu c_\mu^{\nu} = 0,\ \partial_\mu c_\mu^{\nu} = 0\,.$$

Thus we have massless states corresponding to a rank 2 tensor in $D-2$ directions, that is, graviton, K-R, and dilaton, which is the same as closed string.

The interactions do not break these constraints due to the closure of Lie algebra.

Generalized Low energy effective theory

9. Generalized Minimal Model

In principle we can consider any Lie algebra $\mathcal{L}$ instead of the matrix algebra because any Lie algebra is a sub Lie algebra of the matrix algebra:

$$\mathrm{End}(V) \implies \mathcal{L} \subset \mathrm{End}(V)$$

A simple choice of such $\mathcal{L}$ is the set of differential operators of the form:

$$\mathcal{L} = \left\{ v^\mu(x)\partial_\mu + t^a b^a(x), \partial_\mu v^\mu = 0 \right\}.$$

Here t^a are the generators of a finite dimensional Lie algebra.

In other words, $\mathcal{L} = \left\{ v^\mu(x) D_\mu \,,\ \partial_\mu v^\mu = 0 \right\}$.

Here, $D_\mu = \partial_\mu + t^a a_\mu^a(x)$ is the covariant derivative.

Therefore a simple possible low energy effective theory around flat $\mathbb{R}^{10}$ with gauge group G is given by

$$A_\mu = e_\mu{}^\nu(x)(\partial_\nu + t^a a^a_\nu(x))\,, \partial_\nu e^\nu_\mu = 0\,,$$
$$\Psi_s = t^a\,\psi_s{}^a(x)\,.$$

EOM: $\left[A_\nu, \left[A_\mu, A_\nu\right]\right] + \bar{\Psi}\gamma^\mu\Psi = 0\,,$

$$\bar{\Psi}\gamma^\mu[A_\mu, \Psi] = 0\,.$$

Gauge symmetry: $\delta A_\mu = \left[v, A_\mu\right], \quad \delta\Psi = [v, \Psi]\,,$

where $v = v^\mu(x)\partial_\mu + t^a b^a(x),\ \partial_\mu v^\mu = 0.$

This model consists of

YM + Maj-Weyl fermion with adj. rep. + gravity .

SM from generalized Minimal Model

10. SM as a low energy effective theory

We can consider the following back ground for the generalized minimal model:

$$A_\mu^{(0)} = \begin{cases} \dfrac{\partial}{\partial x^m} & (m = 1\sim 4), \\ \dfrac{\partial}{\partial y^i} + t^a a_i^{(0)\,a}(y) & (i = 1\sim 6)\,. \end{cases}$$

$$\Psi_s^{(0)} = 0.$$

Here $x \in \mathbb{R}^4$ and $y \in M_6$, where M_6 is a 6D compact mfd.

Question

Can we realize the SM, especially chiral fermions?

Major difficulty

All fields are adjoint representation. How can we get SM chiral fermions?

Comment: Abe, Kobayashi, Ohki, Oikawa1Sumita

Abe et.al. have considered various models of this type with low energy SUSY.

Here we consider models without SUSY.

11. E8 minimal model

A simple model can be made by considering $G = E8$.

According to $E8 \supset SO(16) \supset SO(10) \otimes SO(6)$,

$$\mathrm{adj}(\mathrm{E8}) = \mathrm{adj}\big(\mathrm{SO}(16)\big) \oplus \mathrm{s}(\mathrm{SO}(16)) \,,$$

$$\begin{aligned} \mathrm{s}\big(\mathrm{SO}(16)\big) = {} & \mathrm{s}\big(\mathrm{SO}(10)\big) \otimes \mathrm{s}\big(\mathrm{SO}(6)\big) \\ & \oplus\, c\big(SO(10)\big) \otimes c(SO(6)) \,. \end{aligned}$$

s: spinor with positive chirality
c: negative

If the b.g. gauge field on M_6 , $t^a a_i^{(0)\,a}(y)$, consists of the above $\mathrm{SO}(6)$ and has index$= 3$, we have 3 generations of 4D chiral spinor with $\mathrm{s}(\mathrm{SO}(10))$, which represents quarks and leptons in SM.

Decomposition of Ψ_s^a

space time index s

According to $SO(9,1) \supset SO(3,1) \times SO(6)$, we have

$$s(SO(9,1)) \\ = s(SO(3,1)) \otimes s(SO(6)) \oplus c(SO(3,1) \otimes c(SO(6)).$$

Therefore,

$$\mathrm{MajWeyl}(SO(9,1)) \cong s(SO(3,1)) \otimes s(SO(6)).$$

In other words, 16 real compts = 2× 4 complex compts.

$$\Psi_s^a \cong \Psi_{\alpha,j}^a \quad s = 1\sim 16 : 10D \text{ Majorana Weyl}$$

$$\alpha = 1\sim 2 : 4D \text{ Weyl} \in s(SO(3,1))$$

$$j = 1\sim 4 : 6D \text{ spinor} \in s(SO(6))$$

internal index a

According to E8⊃ $SO(16)$,

$$\mathrm{adj(E8) = adj(SO(16))} \oplus \mathrm{s(SO(16))}.$$

According to $SO(16) \supset SO(10) \times SO(6)$,

$$\mathrm{s(SO(16)) = s\big(SO(6)\big) \otimes s\big(SO(10)\big)} \\ \oplus \mathrm{c\big(SO(6)\big) \otimes c\big(SO(10)\big)}$$

Therfore, each $\Psi^{a}_{\alpha,j}$ is decomposed as

$$\Psi^{a}_{\alpha,j} \Leftrightarrow \Psi^{B}_{\alpha,j}, \Psi^{k,l}_{\alpha,j}, \Psi^{\bar{k},\bar{l}}_{\alpha,j}$$

$$B = 1{\sim}120 \in \mathrm{adj(SO(16))}$$

$$k = 1{\sim}\ 4 \in s\big(SO(6)\big), l = 1{\sim}\ 16 \in s\big(SO(10)\big).$$

Thus we have

$$\Psi^{k,l}_{\alpha\ j}(x,y) = \sum_{g=1}^{3} \psi_{\alpha}^{(g)l}(x)\, \xi_j^{(g)k}(y)\,,$$

where $\xi_j^{(g)}(y)$ $(g = 1 \sim 3)$ are the zero modes of $D_6^{(0)}$:

$$D_6^{(0)}\, \xi^{(g)} = 0,\ D_6^{(0)} = \gamma^i (\frac{\partial}{\partial y^i} + t^a a_i^{(0)\,a}(y)).$$

Note that t^a are generators of $SO(6)$ so that they act on the $s(SO(6))$ index k.

Therefore, we have 3 generations of $s(SO(10))$.

An example

$M_6 = T_2 \times T_4$

$SO(6) \cong SU(4) \supset SU(3) \times U(1)$

$$t^a a_i^{(0)\,a}(y) = \begin{cases} \mathrm{U(1)\ const.\ mag.\ field\ on}\ T_2 \\ \mathrm{SU(3)\ instanton\ on}\ T_4 \end{cases}$$

$$\mathrm{index} = \frac{1}{3!}\int \mathrm{tr}_{s(SO(6))} F_6^3 = \mathrm{index}_{\mathrm{T}_2} \times \mathrm{index}_{\mathrm{T}_4} = 3$$

Note:

This back ground satisfies EOM of the MM without mass term.

SM Higgs

We can show that SM Higgs doublet can be obtained as a massless mode of the fluctuations of A_i^a around the back ground:

$$A_i^a = A_i^{(0)a} + \phi_i^a \, , (i = 1{\sim}6).$$

In fact, the SM Higgs comes from a vector rep. of $SO(10)$.

According to $\mathrm{SO}(16) \supset \mathrm{SO}(6) \times \mathrm{SO}(10)$,

$$\begin{aligned} \mathrm{adj}(\mathrm{SO}(16)) &= \mathrm{adj}\big(\mathrm{SO}(6)\big) \oplus \mathrm{adj}(\mathrm{SO}(10)) \\ &\oplus \mathrm{vec}(\mathrm{SO}(6)) \otimes \mathrm{vec}(\mathrm{SO}(10)) \, . \end{aligned}$$

Therefore, $\phi_i^a \supset \phi_i^{bc}, b = 1{\sim}6 \in \mathrm{vec}\big(\mathrm{SO}(6)\big)$

$$, \mathrm{c} = 1{\sim}10 \in \mathrm{vec}\big(\mathrm{SO}(10)\big) \, .$$

From the EOM for A_μ we find that the mass of ϕ in 4D is determined by the 6D Laplacian:

$$D_j^2\phi_i - D_i(D_j\phi_j) + 2iF_{ij}\phi_j = -M^2\phi_i \,,$$

where $D_i = \frac{\partial}{\partial y^i} + t^a a_i^{(0)\,a}(y)$ and $F_{ij} = -i[D_i, D_j]$.

Note that $t^a \in SO(6)$ so that it acts on the index b of ϕ^{bc} as a vector representation.

Unlike fermions, there is no index theorem, so eigenvalues are generally nonzero. However, by adjusting the M_6 metric and the Wilson lines, it is possible to make only one (or two) modes have zero mass, while the m^2 of the other modes remains positive.

In this way, we have one (or two) Higgs doublets.

Summary for PART 2

If the emergent spacetime can be described by matrices that fluctuate around diagonal matrices, it is useful to represent the matrices as bi-local fields.

The action of the IIB matrix model in terms of bilocal field can be regarded as a kind of pre-geometric theory.

GR arises as a low energy effective theory consisting of NG bosons around the emergent geometry.

Furthermore, the Standard Model may naturally emerge as the low-energy effective theory of the IIB matrix model. However, it is not yet clear whether the E8 structure is truly realized from the IIB matrix model.

Thank you very much.