Emergent Space-time in QG Analogue of Randomly Connected Tensor Network

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1. Understanding how classical space-time emerges from microscopic quantum DoF remains one of central problems in QG.
2. Randomly Connected Tensor Networks(RCTN) provide an intriguing framework in which continuous geometry arises from purely discrete tensor networks without assuming any background manifold.

3. In this talk, i will introduce a QG analogue of RCTN based on an interacting bosonic many-body system governed by a cubic Hamiltonian.
4. By imposing a constraint: Wheeler-DeWitt equation, we identify the physical Hilbert space of the model and investigate its correlation functions.
5. Different symmetry patterns imposed on the coupling tensor lead to qualitatively different emergent geometries from nothing to fluctuating.
6. These results suggest that symmetry plays a crucial role in determining macroscopic space-time properties and provide a possible route toward understanding emergent geometry from quantum dynamics.

1. Motivation
2. Model
3. Constraint
4. Result
5. Outlook

- Motivation:

$$\text{Quantum Many-body States} \overset{?}{\rightarrow} \text{Emergent Geometry} \underset{?}{\rightarrow} \text{Classical Space-time Structure}$$

  - Why QG?

    Physics currently relies on 2 fundamental, yet mathematically and conceptually incompatible frameworks: GR and QFT.

  - Is space-time fundamental?

    GR: yes, (M, g_{ab}). However, it is widely considered to be an emergent phenomenon from discrete.

  - Can geometry emerge from quantum states?

- Existing approaches
  - Causal Set

    Space-time events are related by a partial order that has the physical meaning of causality relations between space-time events.[1]
  - Canonical Tensor Models(CTM)

    It is a tensor model with P_{abc} and Q_{abc} in the canonical formalism and mimics the structure of the ADM formalism of GR.[2,3]
  - Group Field Theory

    The base manifold is taken to be a Lie group. The field ϕ represents a (D-1)-simplex and the interaction a D-simplex made of D+1 (D-1)-simplexes. Thus, its partition function defines a non-perturbative sum over all simplexes.[4]
  - Randomly Connected Tensor Networks(RCTN)

    Our work is a QG analogue:

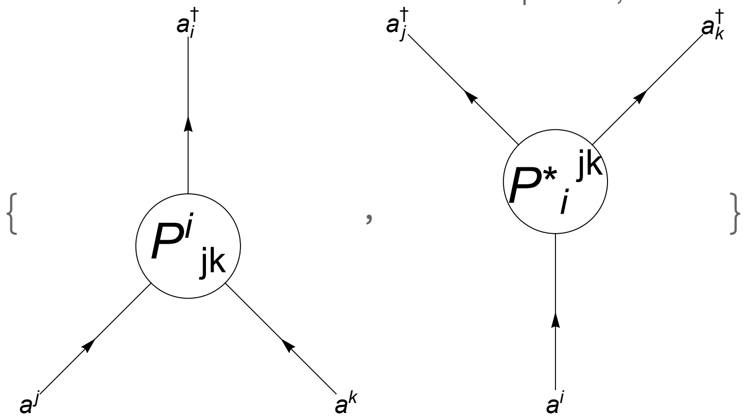
    $$\text{Quantum Many-body States} \rightarrow \text{Hilbert Space} \rightarrow \text{Emergent Geometry}$$

- Model

Hamiltonian:

$$H = P^i_{jk} a^\dagger_i a^j a^k + Q^{jk}_i a^\dagger_k a^\dagger_j a^i, \quad i, j, k = 1, ..., m, \quad Q = P^*$$

where $a^\dagger$ and a are creation and annihilation operators, and tensor P and Q:



with cubic interactions.

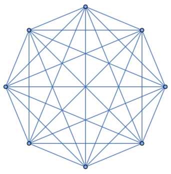
Hilbert space with Fock basis and the physical state:

$$|\vec{n}\rangle = |n_1, ..., n_m\rangle$$

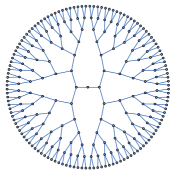
$$|\psi\rangle = \sum C_{\vec{n}}(t)|\vec{n}\rangle$$

- Physical Intuition:

  Tensors at each vertex and sum over all connections:

{"H|ψ⟩≠0:" 

"There are annihilation operator that acts on creation operator of itself.",

"However, what if we put constraint:"  ":H|ψ⟩=0"}

  Classical geometry emerges on the boundary.

- Constraint

  Imposing Wheeler-Dewitt equation:

$$H|\psi\rangle = 0$$

Different choices of tensor give different physical states. The intuition symmetry group(related with space-time) we have in mind is SO(m-1):

$$P^1_{11} = a,$$

$$P^1_{ii} = P^i_{1i} = P^i_{i1} = b, \quad i = 2, 3,$$

$$P^1_{23} = P^2_{13} = P^2_{31} = c = 0 = -P^1_{32} = -P^3_{12} = -P^3_{21},$$

We have assumed *P* is a real and positive tensor with the 1st. direction special. Furthermore, we set:

$$S1 : P^1_{ii} = P^i_{1i} = P^i_{i1} = 1 = b, \quad i = 2, ..., m,$$

$$S2 : P^1_{ii} = 1, \quad i = 2, ..., m,$$

for comparison and others to 0. The symmetry reduces independent tensor components.

- Assumptions:
  - The initial state was chosen such that |2,0,0,...,0⟩th.=1, otherwise the emergence is not obvious.
  - The symmetry we choose is S1, the emergence is also not obvious if not.
  - The constraint equation gives physical states.
- Solution:
  - The physical state $|\psi\rangle$ does not have a simple general solution, so this motivates us to impose symmetry directly on it such that:
    $|\psi\rangle = \sum_{m,n} C_{m,n}(t)(-1)^{n/2+m}\mathcal{N}_{m,n}\left(a_2^{\dagger\,2} + a_3^{\dagger\,2}\right)^m a_1^{\dagger\,n}|0\rangle, \quad n, m = 0, ..., N.$ With (-1)^(n/2+m) and normalization factors.

$$
\begin{aligned}
i\dot{C}_{m,n}(t) = &\sum_{m,n} C_{m+1,n-1}(-1)^{(n-1)/2+m+1}2(m+1)\sqrt{n} + C_{m-1,n+1}(-1)^{(n+1)/2+m-1}2m\sqrt{n+1} \\
&+ C_{m,n+1}(-1)^{(n+1)/2+m}4m\sqrt{n+1} + C_{m,n-1}(-1)^{(n-1)/2+m}4m\sqrt{n} \\
&= 2\sum_{m,n}(-1)^{(n-1)/2+m+1}\left[\left(1+\frac{1}{m}\right)C_{m+1,n-1} - 2C_{m,n-1} + \sqrt{1+\frac{1}{n}}\,(2C_{m,n+1} - C_{m-1,n+1})\right]m\sqrt{n} \\
&= 0.
\end{aligned}
$$

  - The 1st. order solution of PDE:

$$C^{(1)}(m,n) = \frac{\mathcal{C}(n-m)}{\sqrt{m}n^{1/4}}.$$

  - We expect that 2nd. order the large-m,n analytic continuous solution:
    $2(\partial_m + \partial_n)C^{(2)} + (\frac{1}{m} + \frac{1}{2n})C^{(2)} = \frac{C^{(1)}}{8n^2} - (\frac{1}{m} + \frac{1}{2n})\partial_m C^{(1)} - (\frac{1}{2n} - \frac{1}{m})\partial_n C^{(1)}$, which

$$C^{(2)}(m,n) = \frac{-(m+2n)\mathcal{C}(n-m) + 8mn(\mathcal{C} + \mathcal{C}'\ln m)}{8m^{3/2}n^{5/4}}$$

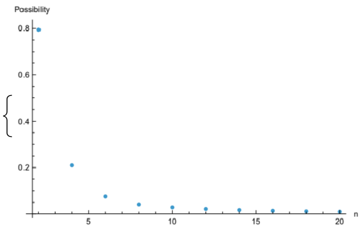
describes propagation behaviours.

- An Analogy of Emergence

  $a^\dagger$ and a decide whether this index's direction exists or not, while P and Q measure the size in this index direction after interactions.

  What observable implies the model we consider with a larger size?

$$\left\{\sum_{fin}\left|\sum_{i}\langle fin|i\rangle\langle i|ini\rangle\right|^{2}=P,\ \left\{\right.\right.$$

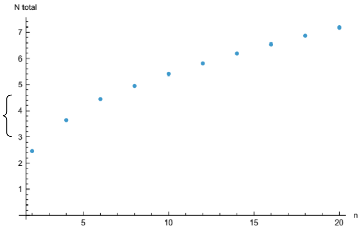


$$\}\}$$

  The probability that the wavefunction of the initial state |ini⟩ falls on the cutoff boundary ⟨fin| after being projected onto the 0-energy subspace.

So the 0-energy subspace is distributed and converged in the internal region without boundary influence as n increases.

$$\left\{ \langle N \rangle = \frac{\sum_{ij} N_{\vec{n},\vec{n}} \langle ini|i\rangle \langle i|\vec{n}\rangle \langle \vec{n}|j\rangle \langle j|ini\rangle}{\sum_k |\langle k|ini\rangle|^2} = \frac{\sum_{ij} N_{\vec{n},\vec{n}} C^*_{i,\vec{n}} C_{j,\vec{n}}}{\sum_k |\langle k|ini\rangle|^2}, \left\{ \right. \right.$$



$$\left. \left. \right\} \right\}$$

The expectation value of the total particle number operator ⟨N⟩ in the 0-energy subspace. So the effective size of the model becomes larger as n increases.

- the effective running dimension $d_{\text{eff}} = \frac{d\text{Log}[\langle N\rangle]}{d\text{Log}[N]} \sim 0.4$.

- Symmetry S1 vs. S2 Comparison

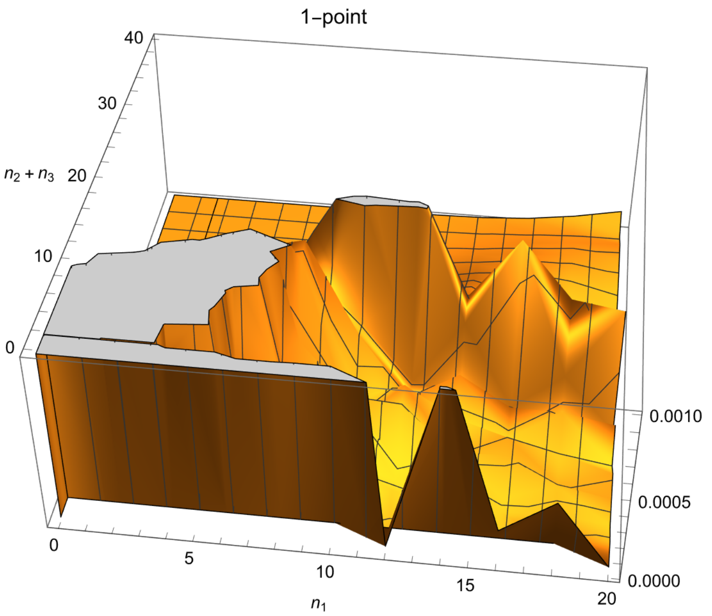
Table 1: $(even\ N) S1$ v.s. $S2 (odd\ N)$

P	$\langle N \rangle$	N	$\dim H$	$\dim \ker H$
0.794239	2.45519	2	27	9
0	3.33333	3	64	24
0.209349	3.63444	4	125	25
0	2.66667	5	216	56
0.075089	4.44685	6	343	49
0	2.66667	7	512	96
0.0412556	4.94462	8	729	81
0	2.66667	9	1000	152
0.028562	5.40308	10	1331	121
0	2.66667	11	1728	216
0.021424	5.81703	12	2197	169
0	2.66667	13	2744	296
0.0167275	6.18267	14	3375	225
0	2.66667	15	4096	384
0.0133994	6.5408	16	4913	289
0	2.66667	17	5832	488
0.0110846	6.86581	18	6859	361
0	2.66667	19	8000	600
0.00933186	7.18021	20	9261	441

$$\dim \ker H = (N+1)^2 (even), \quad \dim \ker H \sim \frac{3}{2} N^2 (odd),$$
$$d = 2,$$
$$\frac{\dim \ker H}{\dim H} = \frac{1}{N+1} \sim \frac{1}{N}.$$

- Result
  - Correlation Functions
    - 1-point of the operator $\hat{O}(\vec{n}) = |\vec{n}\rangle\langle\vec{n}|$:

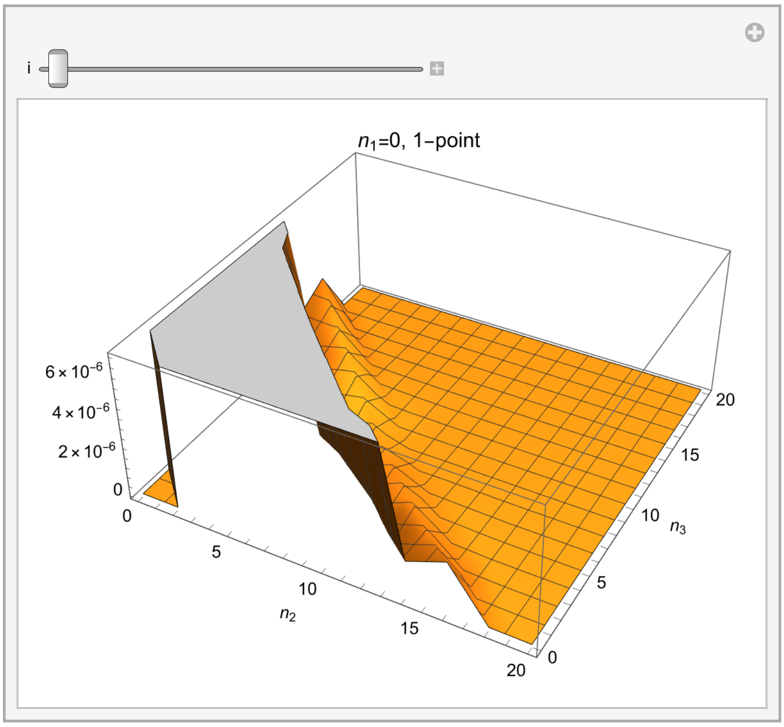
$$\{\langle O(\vec{n})\rangle = \frac{\sum_{ij} C^*_{i,\vec{n}} C_{j,\vec{n}}}{\sum_k |\langle k|ini\rangle|^2}, \{$$



$$\}\}$$

A density operator of number states and describes a quantum system under the certain particle number distribution with the coordinate $(n_1, n_2 + n_3)$.

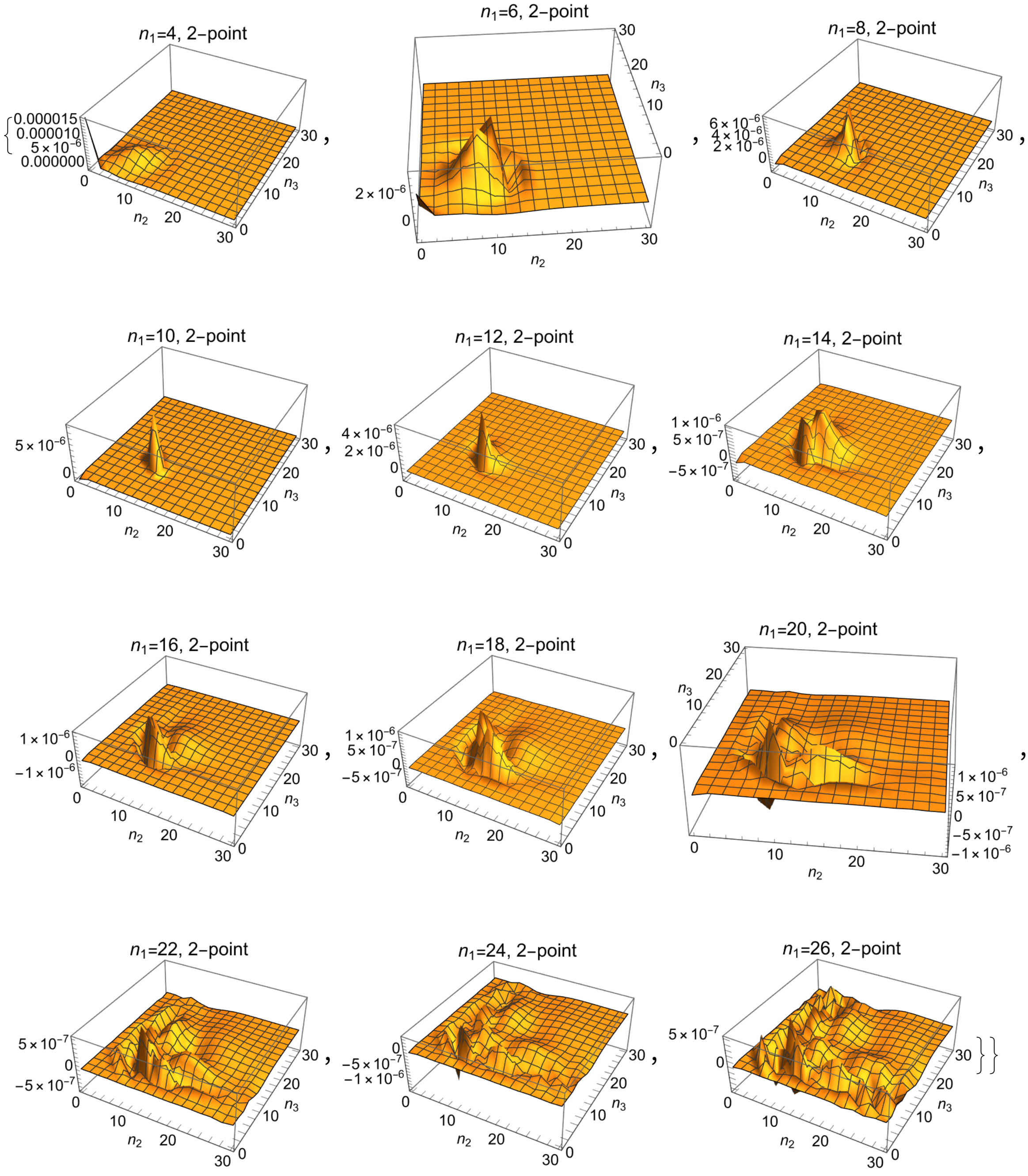
If we interpreted n_1 as the time direction:



- 2-point of the operator $O(\vec{n})KO(\vec{n'}) = O(\vec{n})\sum_k |k\rangle\langle k|O(\vec{n'})$:

Out[∘]=

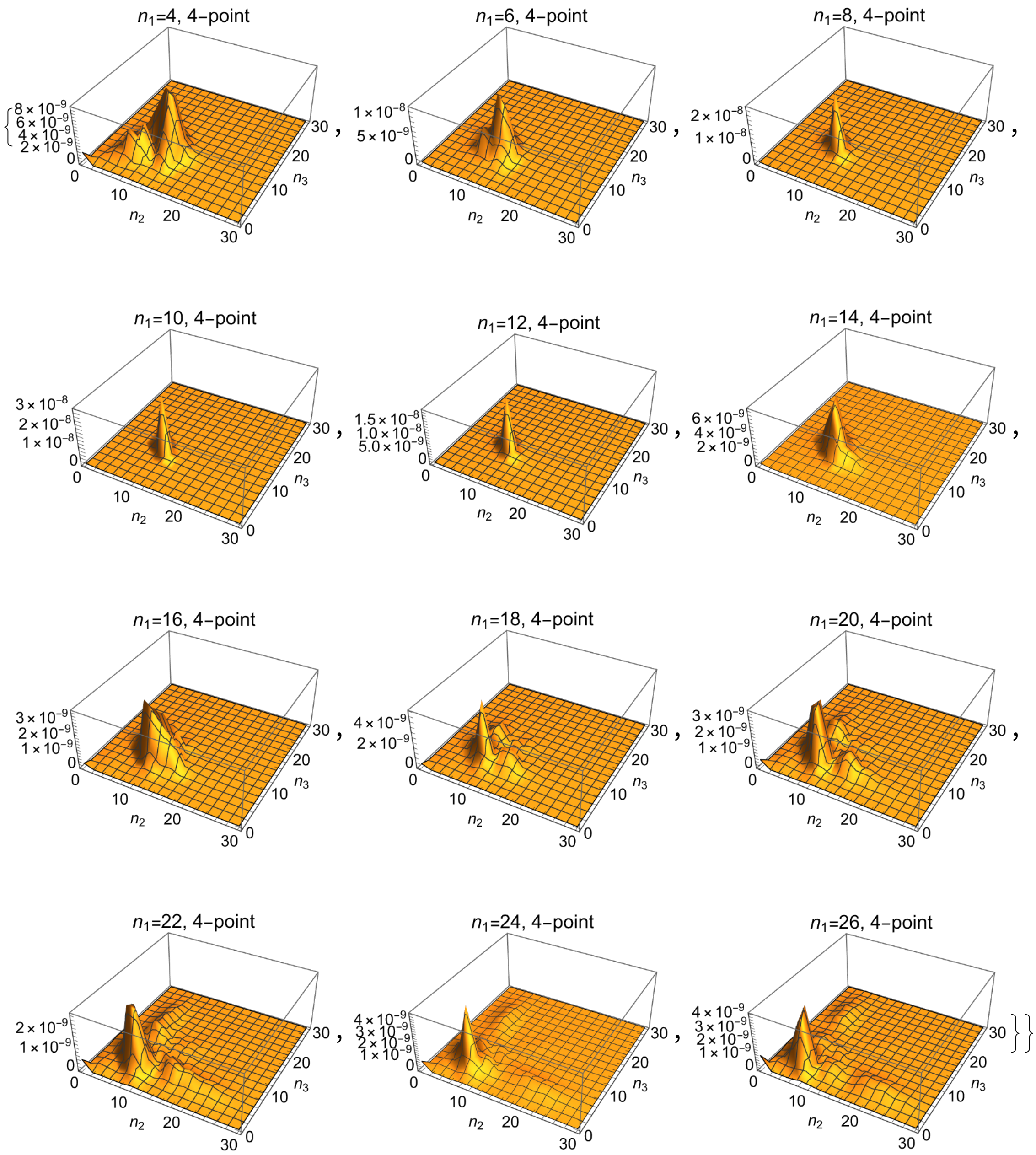
$$\left\{\langle O(\vec{n})KO(\vec{n'})\rangle = \frac{\sum_{ij} C^*_{i,\vec{n}} K^{\vec{n},\vec{n'}} C_{j,\vec{n'}}}{\sum_k |\langle k|ini\rangle|^2},\right.$$



- 4-point of the operator:

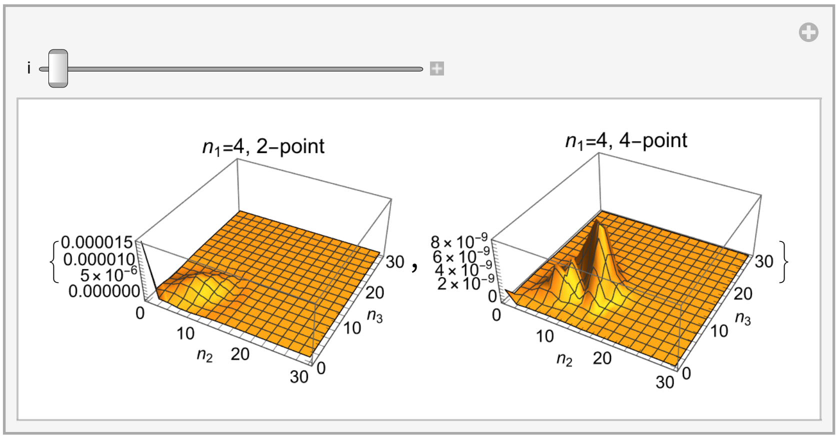
Out[ ]=

$$\left\{ \langle O(\vec{n}) K O(\vec{n'}) K O(\vec{n'}) K O(\vec{n}) \rangle \right. ,$$

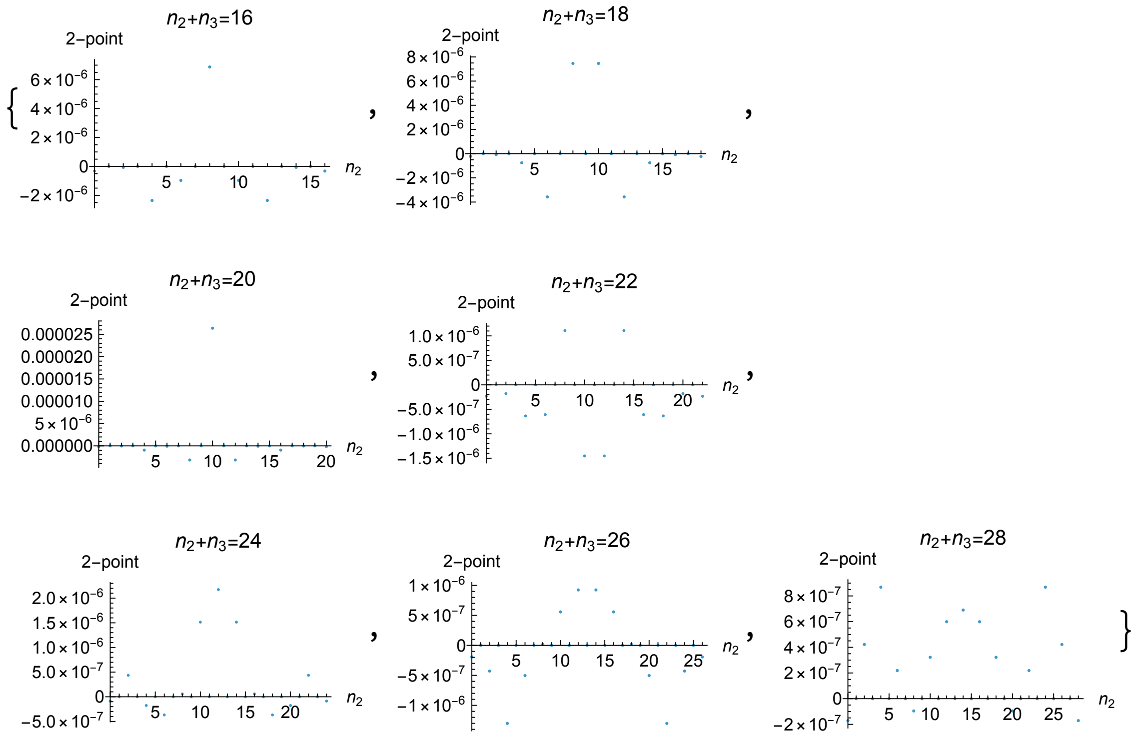


Exciting a source at $\overrightarrow{10}$ in the initial state, evolving by the propagating 0-energy operator K, and finally detecting the amplitude of the excitation at $\vec{n}$: a source starts from $\overrightarrow{10}$, arrives at $\vec{n}$ with self-interaction, and then returns to $\overrightarrow{10}$.

Out[ ]=


i
n_1=4, 2–point
0.000015
0.000010
5×10^{-6}
0.000000
0
10
n_2
20
30
0
10
n_3
20
30
n_1=4, 4–point
8×10^{-9}
6×10^{-9}
4×10^{-9}
2×10^{-9}
0

- Summing over n_1 and to see diagonal structures of (n_2, n_3) where $n_2 + n_3$ is fixed.



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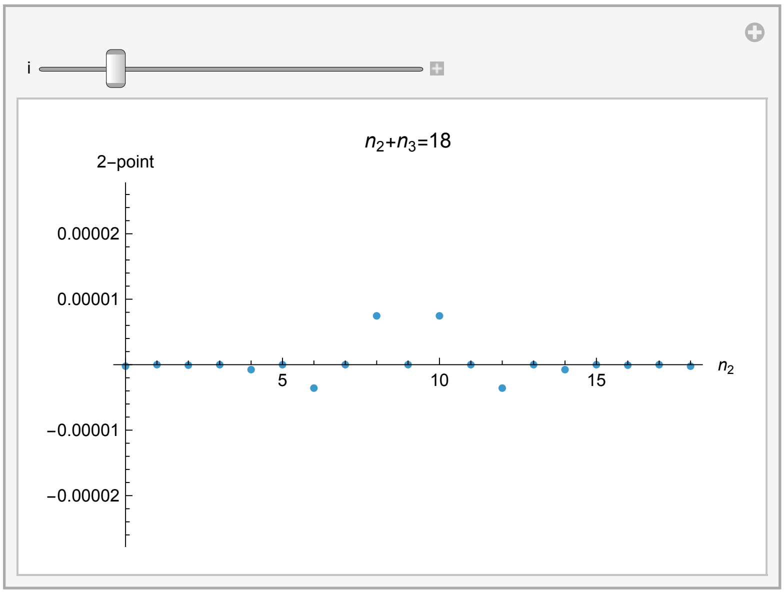
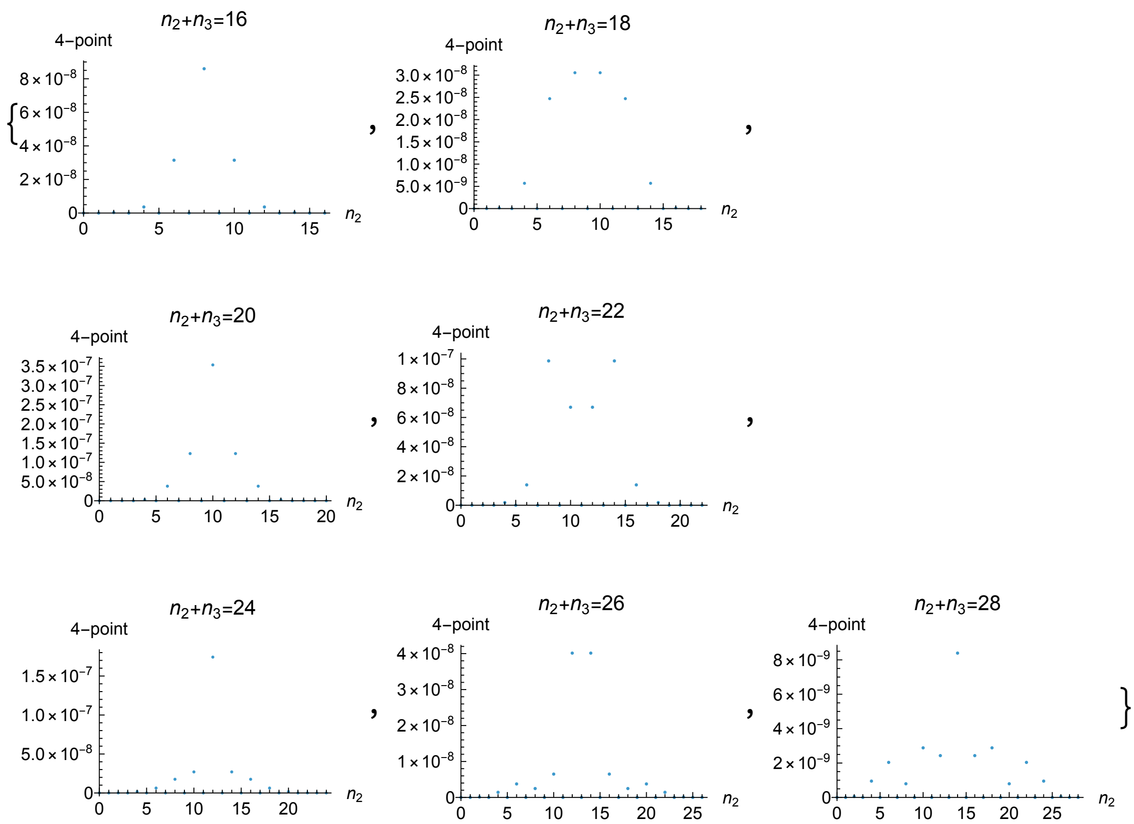
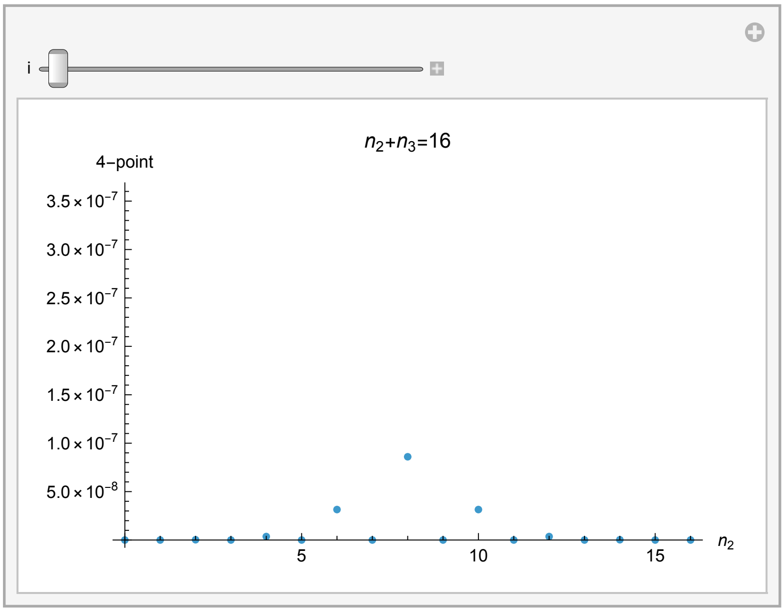
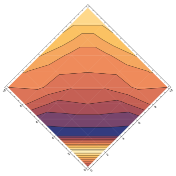


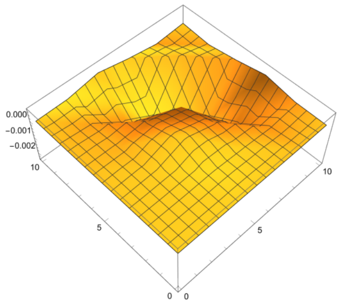

n2+n3=16
4−point
n2
n2+n3=18
4−point
n2
n2+n3=20
4−point
n2
n2+n3=22
4−point
n2
n2+n3=24
4−point
n2
n2+n3=26
4−point
n2
n2+n3=28
4−point
n2

Out[ ]=


i
n2+n3=16
4−point
n2

- Perturbation Influence

$$\left\{\begin{array}{l}|\psi^{(0)}\rangle = K|ini\rangle,\\ |\psi^{(1)}\rangle = |\psi^{(0)}\rangle + \epsilon K|\vec{n'}\rangle\langle\vec{n'}|\psi^{(0)}\rangle,\end{array}\right., \left\{\right.$$  : "It seems no difference"},

{ 

, "Source influence:⟨n1⟩s−⟨n1⟩"},

$$\left.\begin{array}{l}\langle n_1\rangle(n_2,n_3) = \dfrac{\sum_{n_1}\langle\psi^{(0)}|\vec{n}\rangle n_1\langle\vec{n}|\psi^{(0)}\rangle}{\sum_{n_1}|\langle\vec{n}|\psi^{(0)}\rangle|^2},\\ \langle n_1\rangle^s(n_2,n_3) = \dfrac{\sum_{n_1} n_1|\langle\vec{n}|\psi^{(1)}\rangle|^2}{\sum_{n_1}|\langle\vec{n}|\psi^{(1)}\rangle|^2}.\end{array}\right\}$$

- Outlook and Summary
  - First of all, discrete and numerical indexes are too small to say anything about analytic and continuous them definitely. But it gives us some implications.
  - Then, there are lots of assumptions, we still have no idea the reason behind them. A direct generalization is to consider more general complex tensor components.
  - We expect that the max peak would disappear after different correlations with the source(after $n_1 > 10$ on above graphs).
  - Moreover, RCTN may serve as a useful framework for understanding how classical space-time or causality structures can emerge macroscopically from microscopic quantum states and the space-time structure prefers tensor models with symmetric instead of random components specially.
  - At last, the analytic continuous solution may have interesting propagation behaviours at large-m,n.

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Thank you!